Argonne Potential, Three-Body Forces and Stability of Neutron Matter

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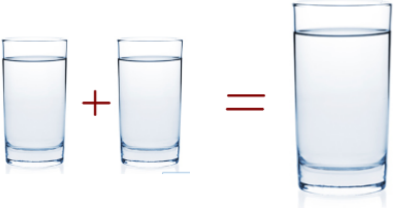
Plan of the Talk

- Stability of ordinary matter
- Stability of nuclear matter
- Argonne potential AV18
- A need in three-body force
- Multineutrons, do they exist?
- Mathematical proof of stability of N-particle systems
- Mathematical proof of instability of neutron matter with AV18+UIX forces

Stability of matter around us

Thermodynamics: the energy is an extensive quantity



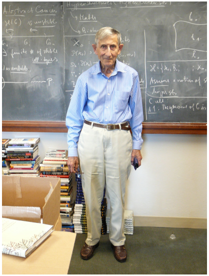
$$\lim_{N\to\infty} E(N)/N = e_0 \quad \text{Exists,}$$

where N is the number of atoms. With pointwise nuclei and electrons $E(N)$ is a mathematically well defined quantity. Stability implies that there exist $c, C > 0$ such that

$$cN \leq E(N) \leq CN$$

Proof of Dyson and Lenard

In 1967 John Freeman Dyson and Andrew Lenard presented a mathematical proof of matter's stability.



Without the Pauli principle the matter is not stable, since without it $E(N) \simeq N^{5/3}$. That is, the assembly of any two macroscopic objects would release energy comparable to that of an atomic bomb.

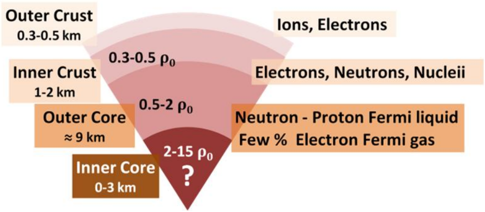
Protons and Neutrons also Form Matter

- For finite nuclei with N neutrons and Z protons ($A = Z + N$)one has Bethe-Weizsäcker formula

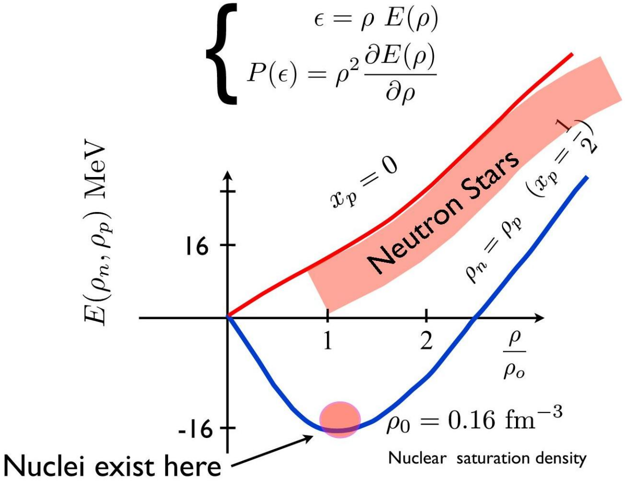
$$E = a_V A - a_S A^{2/3} - a_A \frac{(A-2Z)^2}{A} - \delta(A,Z), \qquad (1)$$

where we assume zero Coulomb forces.

- For large N with gravitational forces we observe neutron stars



Masses and Radii of the Stars are governed ba Equation of State



Nuclear Hamiltonian

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, $m_n - m_p$ effects included

v_{ij}: Argonne v18 (1995)

$$v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{R} + v_{ij}^{CIB}$$

v_{ij}^{γ}: pp, pn & nn electromagnetic terms, Coulomb, magnetic, etc. with form factors

$v_{ij}^{\pi} \sim [Y(r_{ij})\sigma_i \cdot \sigma_j + T(r_{ij})S_{ij}] \otimes \tau_i \cdot \tau_j$; $\langle v_{ij}^{\pi} \rangle$ contributes ∼85% of $\langle v_{ij} \rangle$

$v_{ij}^{R} = \sum_{p=1,14} v_p(r_{ij})O_{ij}^{p}$
$O_{ij}^{p=1,14} = [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2\sigma_i \cdot \sigma_j, (\mathbf{L}\cdot\mathbf{S})^2] \otimes [1, \tau_i \cdot \tau_j]$
Determined phenomenologically

v_{ij}^{CIB}: 4 operators for nuclear charge independence breaking

AV18 is a direct fit to the Nijmegen data base:
1787 pp, 2514 pn, 1 nn data for $E_{Lab} < 350$ MeV ∼40 parameters; χ^2/d.o.f. = 1.09
Typical of modern NN potentials

Three-Body Force

Nucleons are not elementary particles!

Three-body force is **NOT** an iteration of the two-body force

Two-body force

Three-Body Force

Nucleons are not elementary particles!

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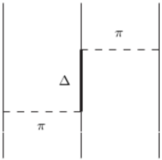
Two-body force

Three-body force

Urbana IX Three-Body Potential

UIX contains **two contributions**

$V^{2\pi}$ – Fujita Myiazawa



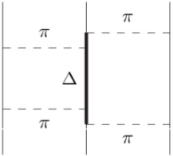
Cyclic sum of three permutations

$$V^{2\pi} = A^{2\pi}(O^{2\pi}_{123} + O^{2\pi}_{231} + O^{2\pi}_{312})$$

$$O^{2\pi}_{123} = \left(\{\hat{X}_{12}, \hat{X}_{23}\}\{\tau_{12}, \tau_{13}\} + \frac{1}{4}[\hat{X}_{12}, \hat{X}_{23}][\tau_{12}, \tau_{23}]\right)$$

$$\hat{X}_{ij} = Y(m_\pi r)\sigma_{ij} + T(m_\pi r)S_{ij}$$

V^R – scalar repulsive term



Cyclic sum of three permutations

$$V^R = U_0 \sum_{cycl} T^2(m_\pi r_{12}) T^2(m_\pi r_{23})$$

Cutoff functions of OPE

Cutoff Functions of OPE

The functions $T(r)$, $Y(r)$ are given through

$$Y(r) = \frac{e^{-\mu r}}{\mu r}\left[1 - e^{-br^2}\right],$$
$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right)\frac{e^{-\mu r}}{\mu r}\left[1 - e^{-br^2}\right]^2.$$

Here $\mu = (m_{\pi_0} + 2m_{\pi_\pm})c/(3\hbar)$ is the average of the pion masses and $b = 2.0\ \mathrm{fm}^{-2}$.

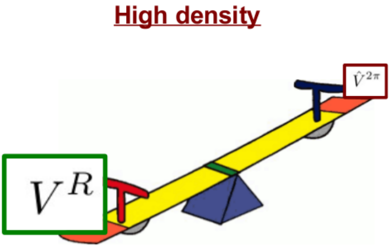
Expanding the exponents it is easy to see that $T(0) = Y(0) = 0$, which means that **the whole three-body interaction vanishes if three nucleons occupy the same position in space!** Urbana VI does not have that problem.

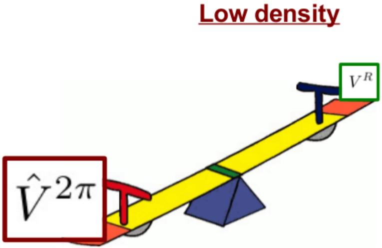
Urbana IX Three-Body Potential

UIX potential has two parameters

- $A^{2\pi}$ adjusted to reproduce the observed binding energies of ^{3}H.
- U_0 tuned for FHNC/SOC calculations to reproduce the empirical equilibrium density of SNM

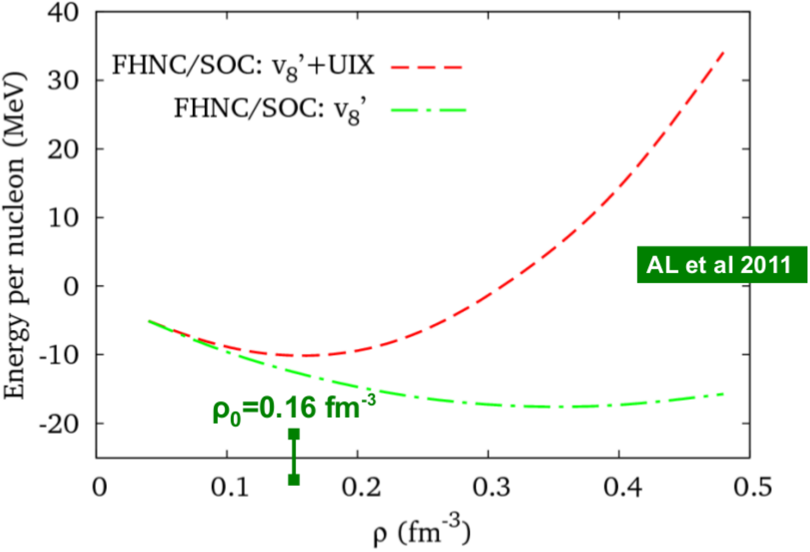
Lagaris and Pandharipande argued that, because of correlations, the relative weight of the contribution depends upon the density of the system:



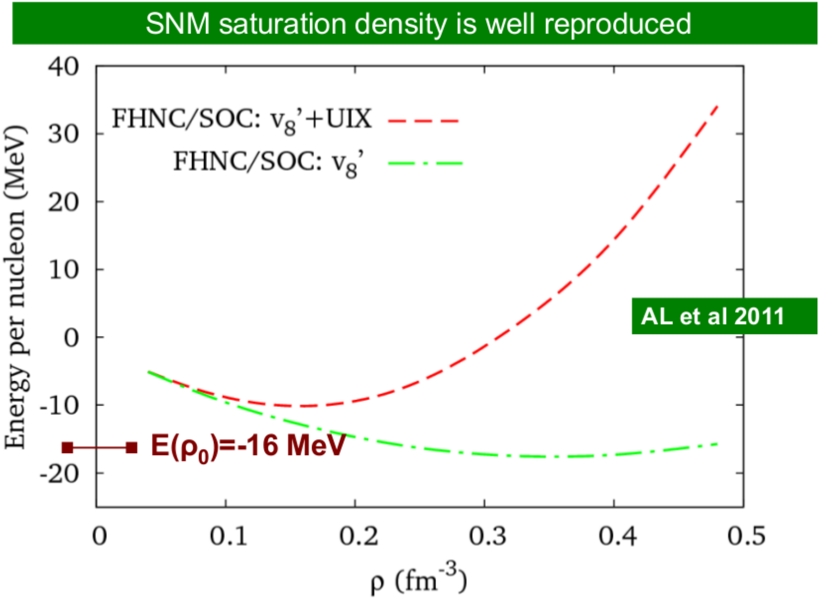


Urbana IX Three-Body Potential

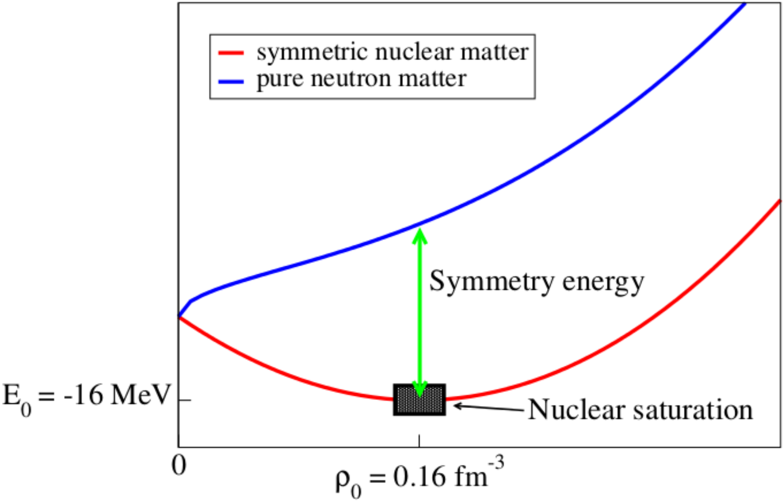
SNM saturation density is well reproduced



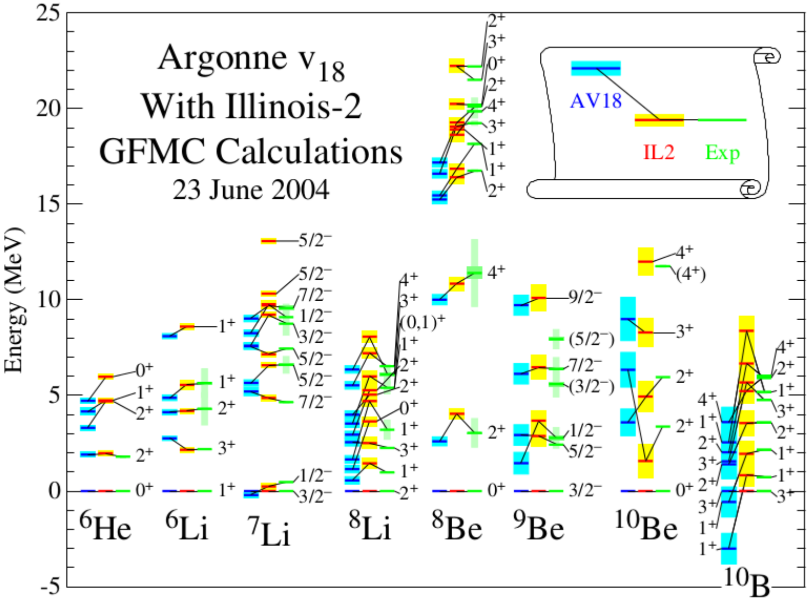
Urbana IX Three-Body Potential



Nuclear Matter vs Pure Neutron Matter



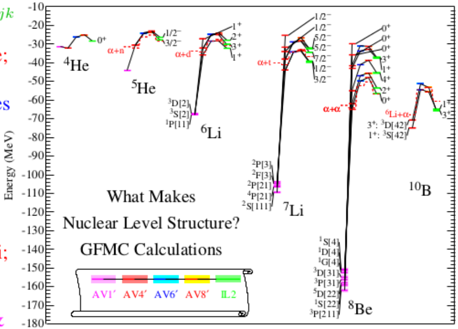
Light Nuclei calculated with AV 18 + Illinois 2



What Makes Nuclear Structure?

Systematically remove features from nuclear Hamiltonian and look at effects on nuclear level energies

- Full AV18 v_{ij} + Illinois V_{ijk} reproduces experiment
- AV8′ - basic level structure; relative stability correct
- AV6′ - Removing $L{\cdot}S$ gives no SO splittings; has A=5,8 gaps but weak A=6,7 binding
- AV4′ - Removing tensor force introduces spurious degeneracies; somewhat overbinds nuclei; A=8 bound
- AV1′ - Pure central force "upside-down" spectrum & no nuclear saturation



Need full, complicated, Hamiltonian to do nuclear physics

The Experiment in which Tetraneutron was “Found”

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Detection of neutron clusters

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A new approach to the production and detection of bound neutron clusters is presented. The technique is based on the breakup of beams of very neutron-rich nuclei and the subsequent detection of the recoiling proton in a liquid scintillator. The method has been tested in the breakup of intermediate energy (30–50 MeV/nucleon) ^{11}Li, ^{14}Be, and ^{15}B beams. Some six events were observed that exhibit the characteristics of a multineutron cluster liberated in the breakup of ^{14}Be, most probably in the channel $^{10}\mathrm{Be}+{}^{4}n$. The various backgrounds that may mimic such a signal are discussed in detail.

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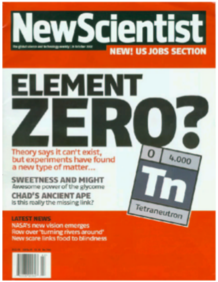


nothing is known [4,5]. The discovery of such neutral systems as bound states would have far-reaching implications for many facets of nuclear physics. In the present paper, the production and detection of free neutron clusters is discussed.

The question as to whether neutral nuclei may exist has a long and checkered history that may be traced back to the early 1960s [5]. Forty years later, the only clear evidence in this respect is that the dineutron is particle unstable. Although ^{3}n is the simplest multineutron candidate, the effects of pairing observed on the neutron drip line suggest that $^{4,6,8}n$ could exhibit bound states [6]. Concerning the tetraneutron, an upper limit on the binding energy of 3.1 MeV is provided by the particle stability of ^{8}He, which does not decay into $\alpha+{}^{4}n$. Furthermore, if ^{4}n was bound by more than 1 MeV, $\alpha+{}^{4}n$ would be the first particle threshold in ^{8}He. As the breakup of ^{8}He is dominated by the ^{6}He channel [7], the tetraneutron, if bound, should be so by less than 1 MeV.

The majority of the calculations performed to date suggest that multineutron systems are unbound [4]. Interestingly, it was also found that subtle changes in the *N*-*N* potentials that do not affect the phase shift analyses may generate bound neutron clusters [5]. In addition to the complexity of such *ab initio* calculations, which include the uncertainties in many-body forces, the n-n interaction is the most poorly known *N*-*N* interaction, as demonstrated by the controversy regarding the determination of the scattering length a_{nn} [8]. The

Tetraneutron in Argonne 18 + Il2 Calculations

AV18 + IL2 does not bind ^{4}n; $E \sim +2.$ MeV

Attempt minimal modifications to AV18+IL2 to give E(^{4}n) ~ -0.5 MeV

Check effects for other nuclei

Modify $^1S_0$ or ^3P_J part of AV18

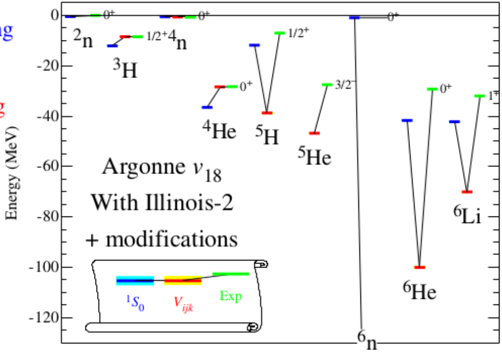
- $^1S_0$ binds ^{2}n
- ^3P_J must be insanely strong

Add $T = \frac{3}{2} V_{ijk}$ attraction

- No effect on NN scattering
- No effect on ^{3}H, ^{3}He, ^{4}He

Add $T = 2V_{ijkl}$ attraction (not shown)

- No effect on ^{3}H, ^{3}He, ^{4}H ^{4}H, ^{5}He, ^{6}Li
- Extreme (GeV scale) binding of ^{5,6,8}n, ^{6}He, etc.



Necessary stability condition for pairwise interacting matter

Theorem [Zhislin, Vugal'ter] Let $E(N)$ denote the ground state energy of N fermions (or bosons) that interact through the pair potential $v(\mathbf{r})$ satisfying the following condition

$$\int_{\mathbf{r}_1,\mathbf{r}_2\in K} v(\mathbf{r}_1-\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2 < 0 \qquad (1)$$

where K is a fixed arbitrary finite cube in $\mathbb{R}^3$. Then

$$E(N) < -cN^2 \qquad \text{for } N > N_0,$$

where $c, N_0 > 0$ are constants. Condition (1) can be improved

$$\int_{\mathbf{r}_1,\mathbf{r}_2\in K_1} v(\mathbf{r}_1-\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2 + \int_{\substack{\mathbf{r}_1\in K_1\\ \mathbf{r}_2\in K_2}} v(\mathbf{r}_1-\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2 < 0,$$

where $K_1 \cap K_2 = \emptyset$ are two disjoint cubes of equal size.

Sketch of the Proof

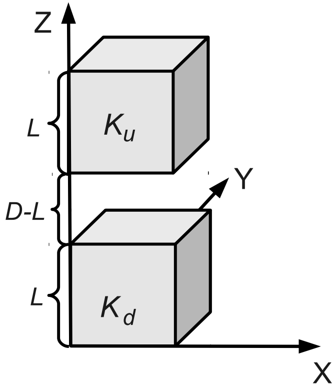


Figure: The neutrons are placed into two disjoint cubes K_u, K_d each with the side length L (subscripts u, d stand for "up" and "down" respectively). The upper cube is shifted by a distance D along the Z-axis with respect to the lower cube.

Sketch of the Proof

Consider $2N$ neutrons that are described by the following Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{2N}\Delta_{\mathbf{r}_i} + V_{2b} + V_{3b} = T + V_{2b} + V_{3b}.$$

The kinetic energy operator T includes the center of mass motion; m is the neutron mass and $\mathbf{r}_i$ for $i = 1, \ldots, 2N$ are neutrons' position vectors. The term $V_{2b} = \sum_{i<j} v_{ij}$ is the sum of two-nucleon interactions given by the Argonne V18 potential. The term V_{3b} is the three–body interaction, which can be one of the modern versions of Illinois three-nucleon interaction, namely Urbana IX or Illinois 7.

Construction of the Trial Function (1/2)

Each cube confines N neutrons, which form an excited state of the Fermi gas. The trial function depends on three parameters $L, D, \omega > 0$, where ω is an integer. For any $p = 1, 2, \ldots$ and $x \in \mathbb{R}$ we set

$$\varphi_p(x) = (L/2)^{-1/2} \sin\left(2\pi p L^{-1} \omega x\right) \quad \text{if } x \in [0, L],$$

and $\varphi_p(x) = 0$ if $x \notin [0, L]$. Let us fix the an integer n in a way that makes the inequality $n^3 \leq N < (n+1)^3$ hold. For each $t = 1, \ldots, N$ we choose a triple of positive integers $\{t_1, t_2, t_3\}$ so that $1 \leq t_1, t_2, t_3 \leq n+1$ and $\sum_{i=1}^{3} |t_i - t_i'| \neq 0$ for $t \neq t'$. Using these triples we define the one particle states for $t = 1, \ldots, N$ as follows

$$f_t(\mathbf{r}) := \varphi_{t_1}(r^x)\varphi_{t_2}(r^y)\varphi_{t_3}(r^z),$$

where r^x, r^y, r^z are the Cartesian components of the vector $\mathbf{r}$.

Construction of the Trial Function (2/2)

Let us set

$$\Psi_{\Pi}(\mathbf{r}_1, \dots, \mathbf{r}_{2N}) := f_1(\mathbf{r}_1) f_2(\mathbf{r}_2) \cdots f_N(\mathbf{r}_N)$$
$$\times f_1(\mathbf{r}_{N+1} - \mathbf{D}) f_2(\mathbf{r}_{N+2} - \mathbf{D}) \cdots f_N(\mathbf{r}_{2N} - \mathbf{D}),$$

where $\mathbf{D} := (0, 0, D)$ is a three-dimensional vector. Let , whose elements $g \in \mathbb{S}_{2N}$ permute only the spatial coordinates. We construct the trial function for $2N$ neutrons as

$$\tilde{\Psi}_A = \Psi_A(\mathbf{r}_1, \dots, \mathbf{r}_{2N}) |n \uparrow\rangle |n \uparrow\rangle \cdots |n \uparrow\rangle,$$

where the spatial part of the wave function is $\Psi_A = \sqrt{(2N)!} \mathcal{A} \Psi_{\Pi}$. Here $\mathcal{A}$ is an antisymmetrizer on the permutation group for $2N$ particles $\mathbb{S}_{2N}$ (only spatial coordinates are permuted).

Poof (continued)

By the variational principle

$$E(2N) \le \langle \tilde{\Psi}_A | T | \tilde{\Psi}_A \rangle + \langle \tilde{\Psi}_A | V_{2b} | \tilde{\Psi}_A \rangle + \langle \tilde{\Psi}_A | V_{3b} | \tilde{\Psi}_A \rangle,$$

where $E(2N)$ is the ground state energy of $2N$ neutrons.
Contribution of kinetic energy is easy to compute

$$\langle \tilde{\Psi}_A | T | \tilde{\Psi}_A \rangle \le \left(\frac{2\pi\omega\hbar}{\sqrt{m}L} \right)^2 \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} \sum_{k=1}^{n+1} (i^2 + j^2 + k^2) = \mathcal{O}(N^{5/3})$$

Contribution of the 2-body interactions

$$\langle \tilde{\Psi}_A | V_{2b} | \tilde{\Psi}_A \rangle = \mathcal{O}(N^{8/3})$$

The power is different from 2 due the terms proportional to the square of the angular momentum.

3-body Term Contribution

$$\langle \tilde{\Psi}_A | V_{3b} | \tilde{\Psi}_A \rangle = QN^3 + \Upsilon(\omega) N^3$$

Let $W(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ be the 3-body interaction potential. Then the constant Q is defined through

$$Q = \int_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \in K_d} W(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 + \int_{\substack{\mathbf{r}_1, \mathbf{r}_2 \in K_u \\ \mathbf{r}_3 \in K_d}} W(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3$$

Constant Q can be made negative while $\Upsilon(\omega) \to 0$ for large ω due to oscillations in one particle wave functions. Thus the total energy of $2N$ neutrons grows like $-QN^3$. The matter cannot be formed!

Behavior of the 3-body Potential

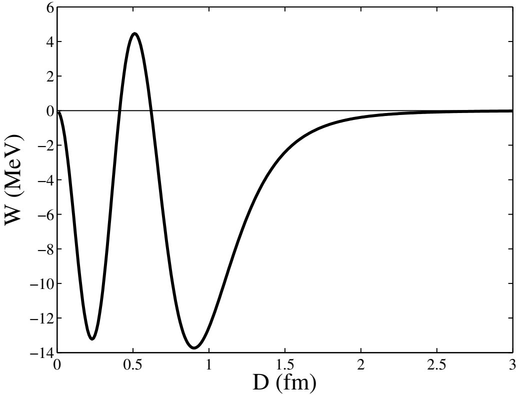


Figure: The plot of the function $W(0, 0, \mathbf{D})$ (where $\mathbf{D} \equiv (0, 0, D)$) versus parameter D. The plot shows that to ensure neutron matter collapse one can set $D = 1$ fm.

Conclusions

- Neutron matter with AV18+UIX forces is unstable: the binding energy of N neutrons in the ground state $E(N)$ grows proportionally to N^3
- The problem is due to the force vanishing when 3 nucleons occupy the same site in space
- Old Urbana VI three-body force does not have that problem
- The matter can be quasistable, one has to calculate the potential barriers and probabilities for their penetration
- Primitive Minnesota and Volkov types of interaction predict bound multineutrons with growing density: the matter is unstable in this case as well since $|E(N)| \simeq N^2$.
- Rigorous mathematical methods provide useful insight into the structure of nuclear forces