INFRARED REGULARIZATION of NONABELIAN GAUGE FIELDS

BEYOND PERTURBATION THEORY.

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Infrared properties of the Yang-Mills field remain rather obscure.

Perturbative scattering matrix does not exist in the Yang-Mills theory, and the only sensible objects are the correlation functions of the gauge invariant operators, or gauge invariant infrared regularized theory. Simple infrared regularization by introducing a mass for the vector field, breaks the gauge invariance of the theory and the limit $m \to 0$ does not exist. On the other hand in the Higgs model the limit $m \to 0$ does exist, but produces a different theory, describing not only the massless vector field but also a scalar particle.

In fact even the quantization of nonabelian gauge fields beyond perturbation theory is an unsolved problem (Gribov ambuguity).

It was discussed in the papers (A.A.Slavnov, Theoretical and Mathematical Physics 154(2008)213, A.A.Slavnov, JHEP 08(2008)047, A.Quadri, A.A.Slavnov JHEP 07(2010)087.) that impossibility to select a unique Lorentz invariant gauge beyond the perturbation theory is not the intrinsic property of the Yang-Mills model, but is related to its particular formulation. Adding new excitations which decouple asymptotically it is possible to quantize nonabelian gauge models in a manifestly Lorentz invariant way both in perturbation theory and beyond it.

Having that in mind we propose to use for the gauge invariant infrared regularization of the Yang-Mills theory the following Lagrangian

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - m^{-2}(D^2\tilde{\phi})^*(D^2\tilde{\phi}) + (D_\mu e)^*(D_\mu b) + (D_\mu b)^*(D_\mu e) \\ +\alpha^2(D_\mu\tilde{\phi})^*(D_\mu\tilde{\phi}) - \alpha^2 m^2(b^*e + e^*b) \quad (1)$$

where ϕ is a two component complex doublet, and

$$\tilde{\phi} = \phi - \widehat{\mu}; \quad \widehat{\mu} = (0, \mu\sqrt{2}g^{-1}) \quad (2)$$

μ is an arbitrary constant which in the following we put equal to m, to simplify the discussion of unitarity. The parameter α goes to zero, when the infrared regularization is removed.

The Lagrangian (1) is obviously invariant with respect to "shifted"gauge transformations:

$$\begin{aligned} A^a_\mu \to A^a_\mu + \partial_\mu \eta^a - g\epsilon^{abc} A^b_\mu \eta^c \\ \phi^a \to \phi^a + \mu \eta^a + \ldots \end{aligned} \tag{3}$$

The field ϕ^a is shifted by an arbitrary function, therefore one can put $\phi^a = 0$. This gauge is algebraic, but Lorentz invariant. It may be used beyond perturbation theory as well.

This Lagrangian is also invariant with respect to the supersymmetry transformations

$$
\begin{aligned}
\phi &\longrightarrow \phi - ib\epsilon \\
e &\longrightarrow e - \frac{D^2(\phi - \widehat{\mu})}{m^2}\epsilon \\
b &\longrightarrow b
\end{aligned}
\tag{4}
$$

where ϵ is a constant Hermitean anticommuting parameter. This symmetry plays a crucial role in the proof of decoupling of unphysical excitations.

In the case under consideration the massive theory with $\alpha \neq 0$ is gauge invariant but not unitary. It may seem strange as usually the gauge invariance is a sufficient condition of unitarity, because one can pass freely from a renormalizable gauge to the unitary one, where the spectrum includes only physical excitations. In the present case there is no "unitary"gauge. Even in the gauge $\phi^a = 0$, there are unphysical excitations.

Invariance of the Lagrangian (1) with respect to the gauge transformation (3) and the supersymmetry transformations (4) makes the effective Lagrangian invariant with respect to the simultaneous BRST transformations corresponding to (3) and the supersymmetry transformations (4). The effective Lagrangian may be written in the form

$$L_{ef} = L + s_1 \overline{c}^a \phi^a = L(x) + \lambda^a \phi^a - \overline{c}^a (\mu c^a - b^a) \tag{5}$$

One can integrate over $\bar{c}, c$ in the path integral determining expectation value of any operator corresponding to observable. It leads to the change $c^a = b^a \mu^{-1}$. After such integration the effective Lagrangian becomes invariant with respect to the transformations which are the sum of the BRST transformations and the supersymmetry transformations (4) with $c^a = b^a \mu^{-1}$. For the asymptotic theory these transformations acquire the form

$$\begin{aligned}
\delta A_\mu^a &= \partial_\mu b^a \mu^{-1} \epsilon \\
\delta \phi^a &= 0 \\
\delta \phi^0 &= -b^0 \epsilon \\
\delta e^a &= \partial_\mu A_\mu^a \mu^{-1} \\
\delta e^0 &= -\partial^2 \phi^0 \mu^{-2} \\
\delta b^a &= 0 \\
\delta b^0 &= 0.
\end{aligned} \tag{6}$$

According to the Neuther theorem the invariance with respect to the transformations (6) generates a conserved charge Q, and the physical asymptotic states may be chosen to satisfy the equation

$$\hat{Q}_0|\psi>_{as}=0 \tag{7}$$

where

$$Q_0=\int d^3x[(\partial_0 A_i^a-\partial_i A_0^a)\mu^{-1}\partial_i b^a-\mu^{-1}\partial_\nu A_\nu^a\partial_0 b^a+\mu^{-2}\partial^2(\partial_0\phi^0)b^0-\mu^{-2}\partial_0 b^0\partial^2(\phi^0)-\mu\alpha^2 b^a A_0^a] \tag{8}$$

Due to the conservation of the Neuther charge this condition is invariant with respect to dynamics.

We want to prove that the Lagrangian(5) really describes the infrared regularization of the Yang-Mills theory. That means for $\alpha \neq 0$ it corresponds to a massive gauge invariant theory and in the limit $\alpha = 0$ it describes the usual three dimensionally transversal excitations of the Yang-Mills field. Of course for $\alpha \neq 0$ the spectrum includes also some unphysical excitations.

This regularization differs by the fact that it preserves the manifest gauge invariance of the theory. Even for $\alpha \neq 0$ the theory we consider is gauge invariant.

In the limit $\alpha = 0$ the last term in the eq.(8) disappears. The terms depending only on the fields A_0, A_i and their time derivatives coincide with the usual BRST charge for the Yang-Mills field in the diagonal Feynman gauge provided one identifies the anticommuting fields b_a with the Faddeev-Popov ghost field c_a. The fields b_a, e_a play the role of the Faddeev-Popov ghosts.

By the usual arguments the longitudinal and temporal components of the Yang-Mills field as well as the fields b_a, e_a decouple, and the physical states may include only transversal components of the Yang-Mills field and variables corresponding to the fields ϕ^0, b^0, e^0. One can show that the fields ϕ^0, b^0, e^0 also decouple.

Conclusion

1. Using the alternative formulation of nonabelian gauge theories, described above, one can avoid the Gribov ambiguity and quantize the Yang-Mills theory both in the framework of perturbation theory and beyond it.

2. This formulation allows also to construct a manifestly gauge invariant procedure of infrared regulatization.

3. This approach opens an interesting possibility to describe solitons in the nonabelian gauge theories without physical scalar particles.