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Relations between collinear and K_T factorizations

talk based on results obtained in collaboration with
M. Greco and S.I. Troyan

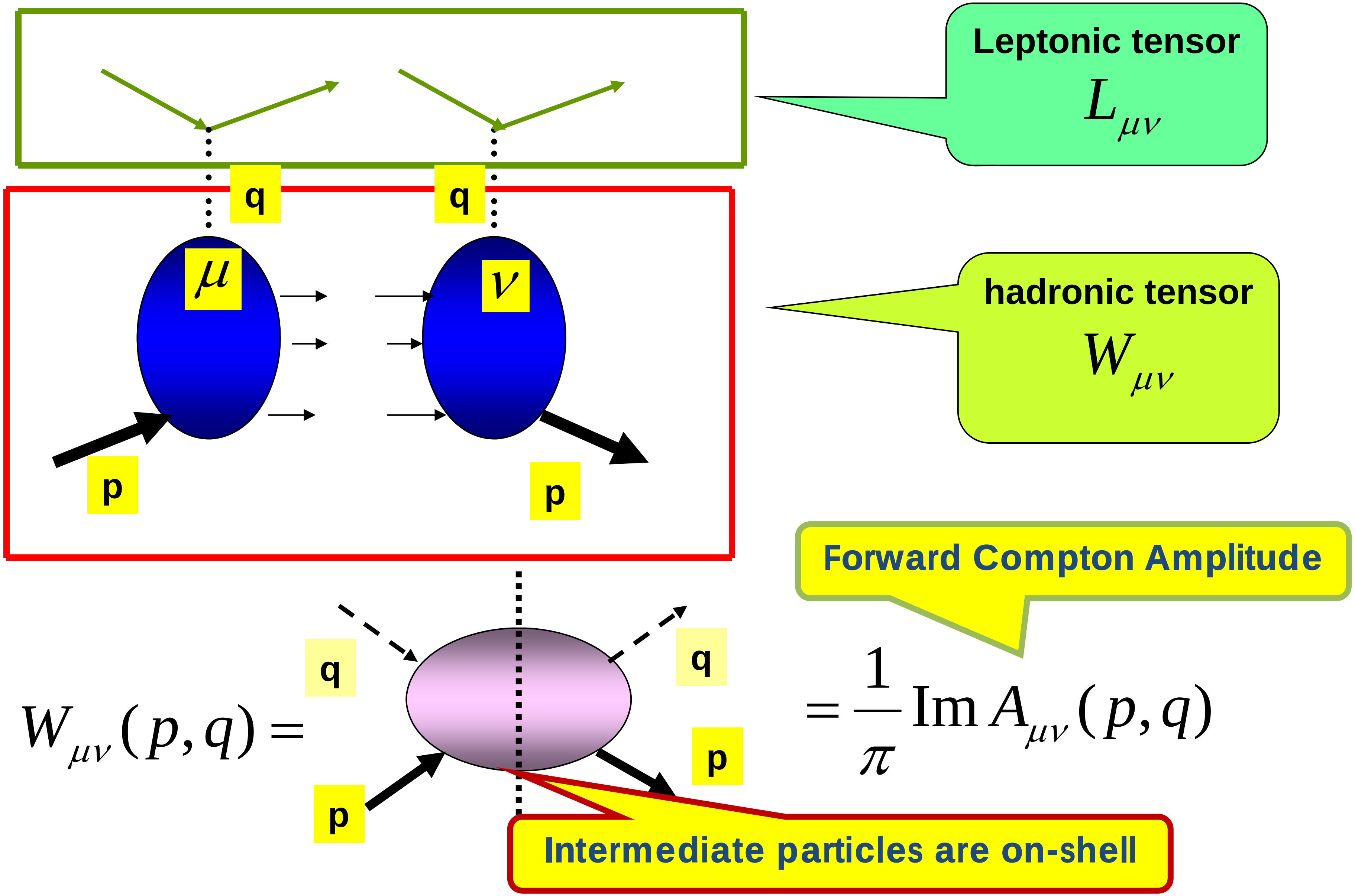
Factorization is one of key concepts in QCD . It has been used to apply perturbative QCD to description of hadronic reactions

The reason: QCD is poorly known (does not exists as a regular science) in the infrared region (at large distances), so lack of such knowledge should be approximated/mimicked somehow and the most popular way to do it is QCD Factorization

For simplicity, I focus on the simple and at the same time important example of hadronic reaction:

Deep-Inelastic lepton-hadron Scattering

DIS Inclusive cross-section:



$$W_{\mu\nu}(p,q) = \quad = \frac{1}{\pi}\,\mathrm{Im}\,A_{\mu\nu}(p,q)$$

For instance, Hadronic tensor for unpolarized electron-proton DIS is conventionally parameterized as follows:

Projection operators

$$q_\mu W_{\mu\nu} = q_\nu W_{\mu\nu} = 0$$

$$W_{\mu\nu}^{unpol} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \left(p_\mu - q_\mu \frac{pq}{q^2} \right) \left(p_\nu - q_\nu \frac{pq}{q^2} \right) \frac{F_2(x, Q^2)}{pq}$$

Structure functions

Arguments of F_1, F_2

$$Q^2 = -q^2 > 0, \quad x = Q^2 / 2pq, \quad 0 < x < 1$$

We focus on the region of small x: x<<1

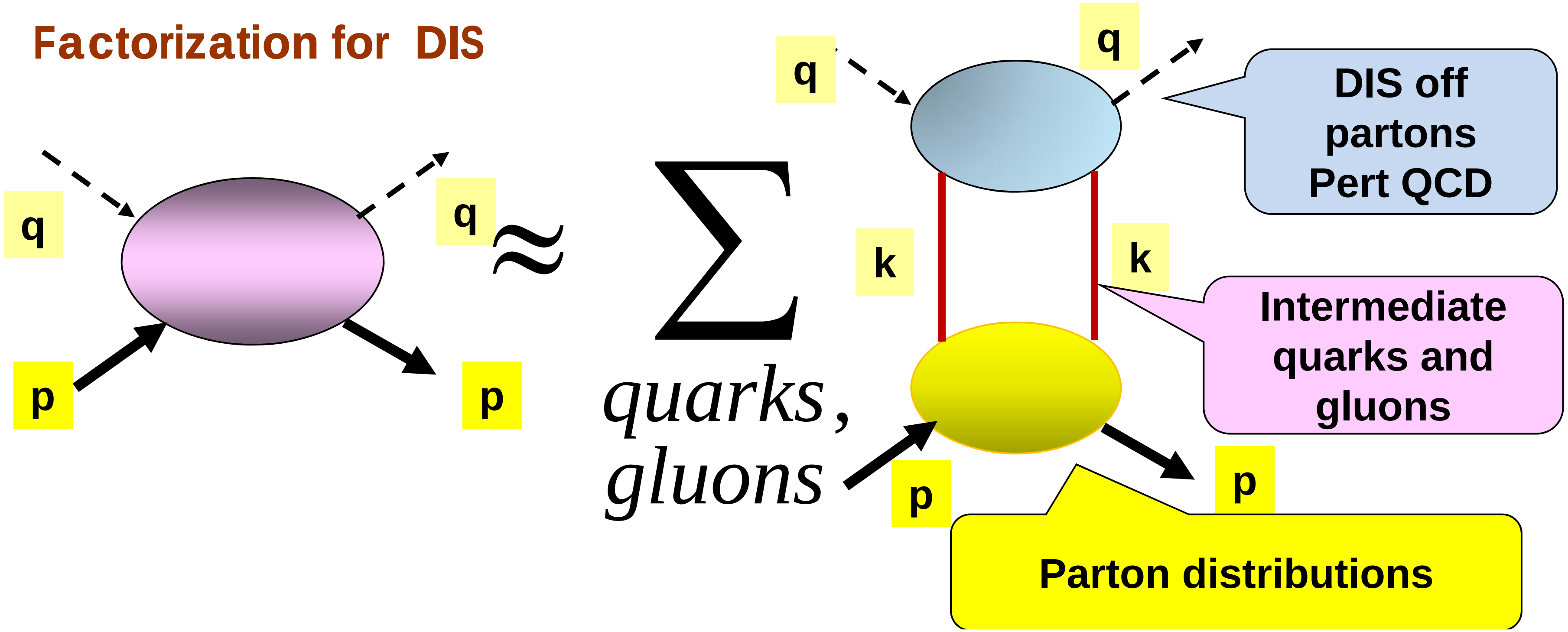
In order to calculate structure functions, one should know both Perturbative and Non-Perturbative QCD but Non-Perturbative QCD is known poorly, so straightforward calcularton of structure functions cannot be performed

Instead of straightforward calculations there is conventionally used approximation of

FACTORIZATION

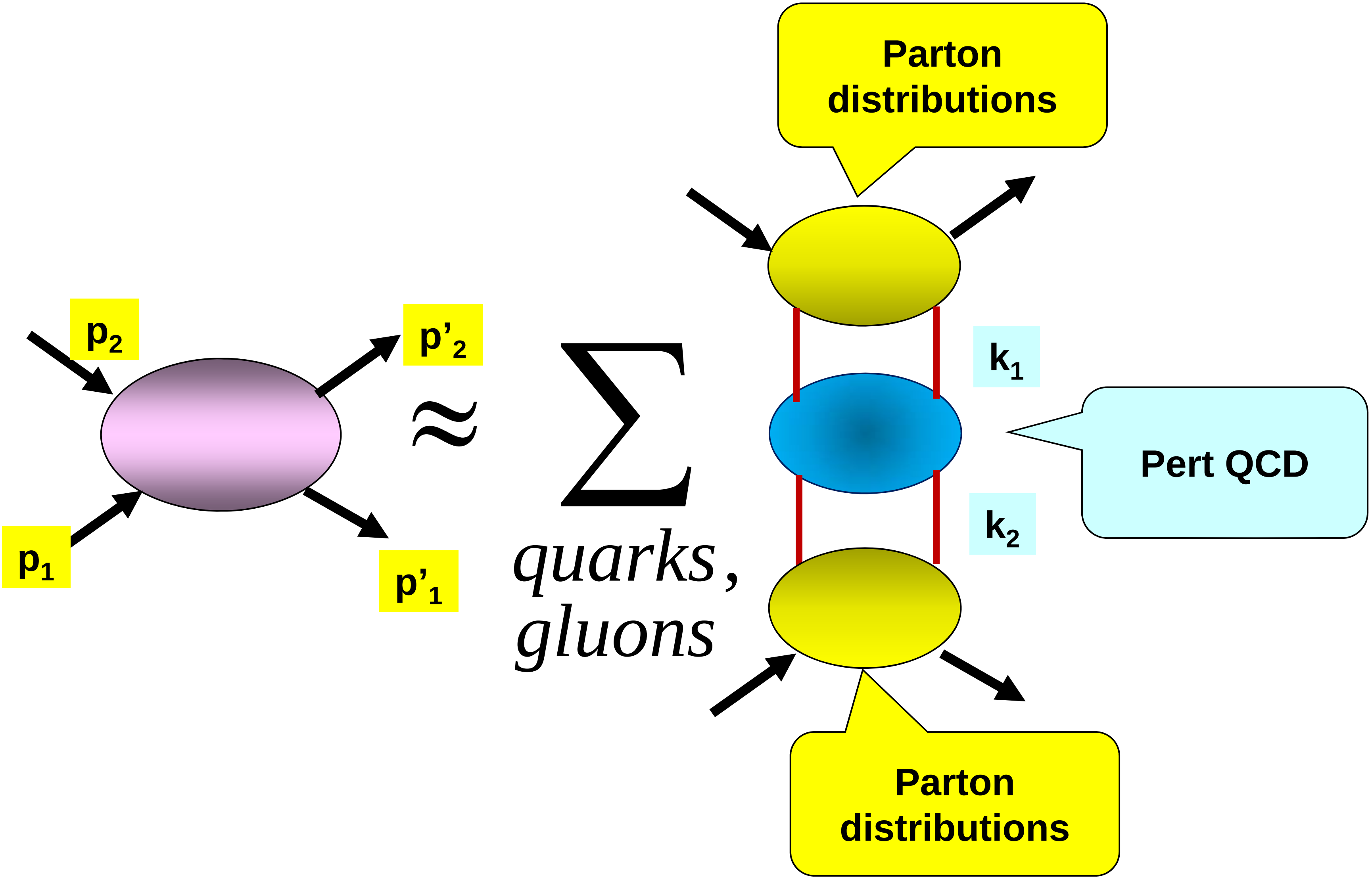
Factorization in general is the principal concept in order to apply Perturbative QCD to description of all hadronic reactions

Factorization for DIS



Parton distributions are usually found from phenomenological considerations I will present theoretical restrictions on them

Similarly, amplitudes of 2 ->2 hadronic reactions are represented through factorization convolutions



There are known the following kinds of factorization in the literature:

Collinear Factorization

Amati-Petronzio-Veneziano, Efremov-Ginzburg-Radyushkin, Libby-Sterman, Brodsky-Lepage, Collins-Soper-Sterman

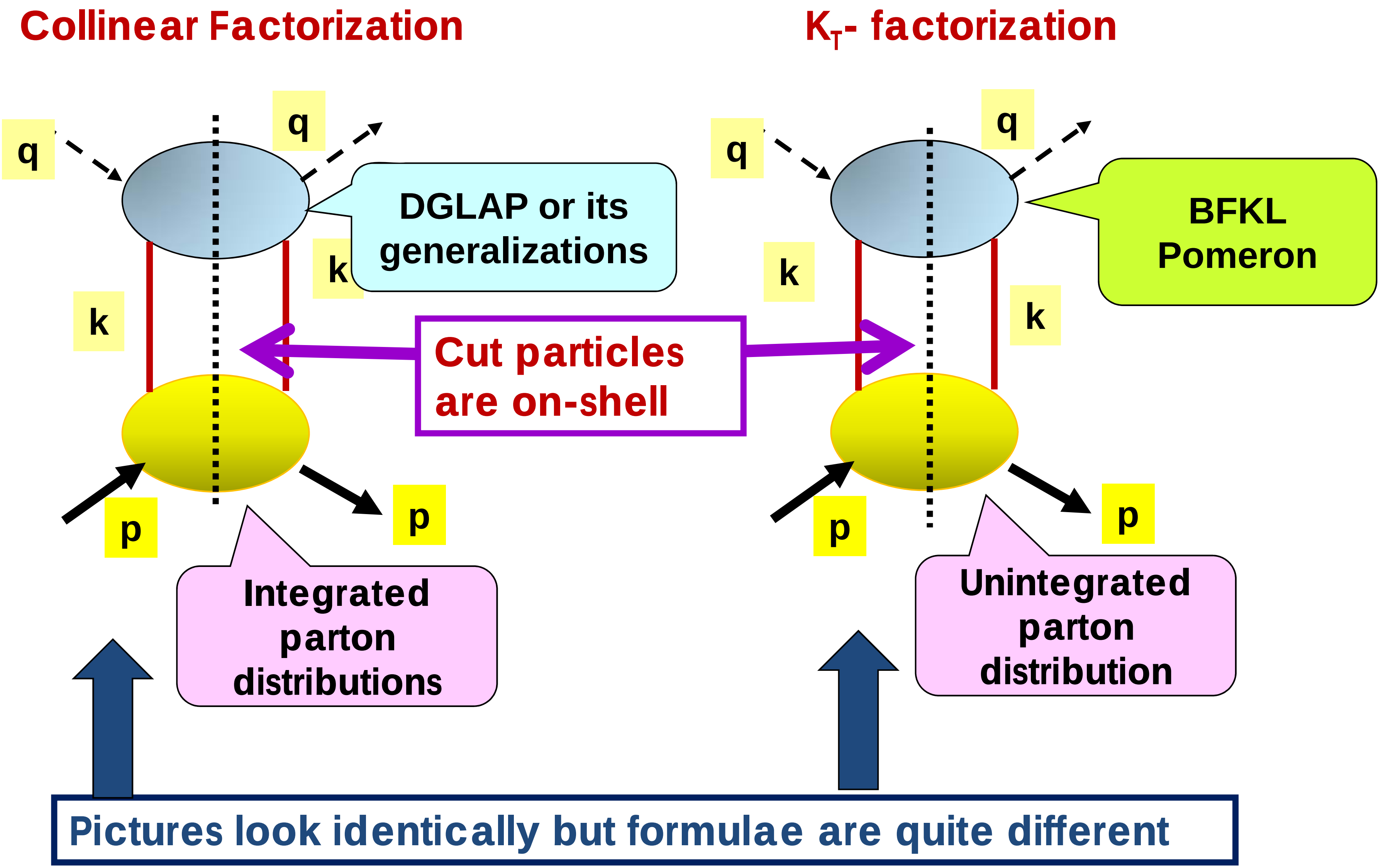
K_T- factorization

S. Catani - M. Ciafaloni – F. Hautmann

These kinds of factorization were introduced from different considerations and are used for different perturbative approaches, so they look absolutely unrelated to each other.

I will show they are related

Conventional illustrations of Factorizations



Factorization representations for DIS structure functions

fraction of longitudinal moment

factorization scale

Collinear Factorization

$$f(x,Q^2) = \int_x^1 \frac{d\beta}{\beta} f^{(pert)}(x/\beta, Q^2/\mu^2)\,\varphi(\beta,\mu^2)$$

integrated parton distribution

Perturbative terms are calculated with evolution equations

Parton distributions found from phenomenological considerations

K_T-Factorization

$$f(x,Q^2) = \int \frac{d\beta}{\beta}\frac{dk_\perp^2}{k_\perp^2} f^{(pert)}(x/\beta, Q^2/k_\perp^2)\,\Phi(\beta,k_\perp^2)$$

Unintegrated parton distribution

Perturbative contents of structure functions are often represented through Mellin transform. **For instance, in collinear factorization**

$$f^{(pert)}(x,Q^2) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i}\; x^{-\omega} C(\omega) \exp\left[\int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\gamma(\omega,\alpha_s)\right]$$

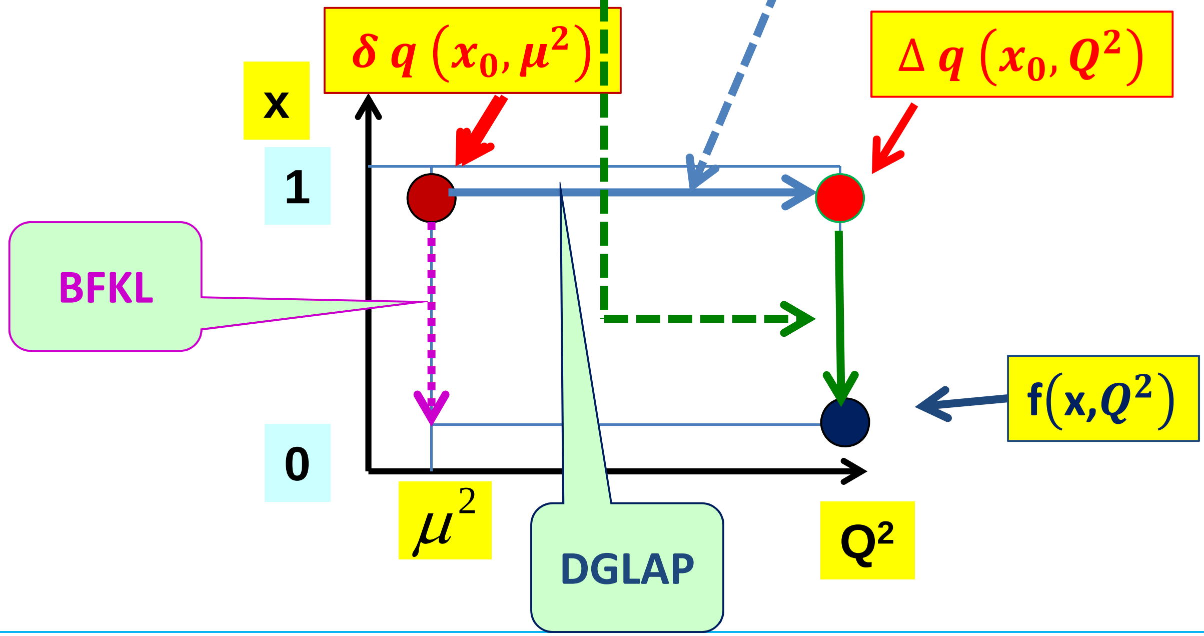
Coefficient function controls the x-evolution

Anomalous dimension controls the Q^2 -evolution

Calculation of coeff functions and anom dims = calculation and summation of involved Feynman graphs - Pert QCD
There are known methods for total resummations: e.g. BFKL, DGLAP and its generalizations to the small-x region

$$f(x,Q^2) = C(x,x_0) \otimes \Delta q(x_0,Q^2) = C(x,x_0) \otimes E(Q^2,\mu^2) \otimes \delta q(x_0,\mu^2) =$$

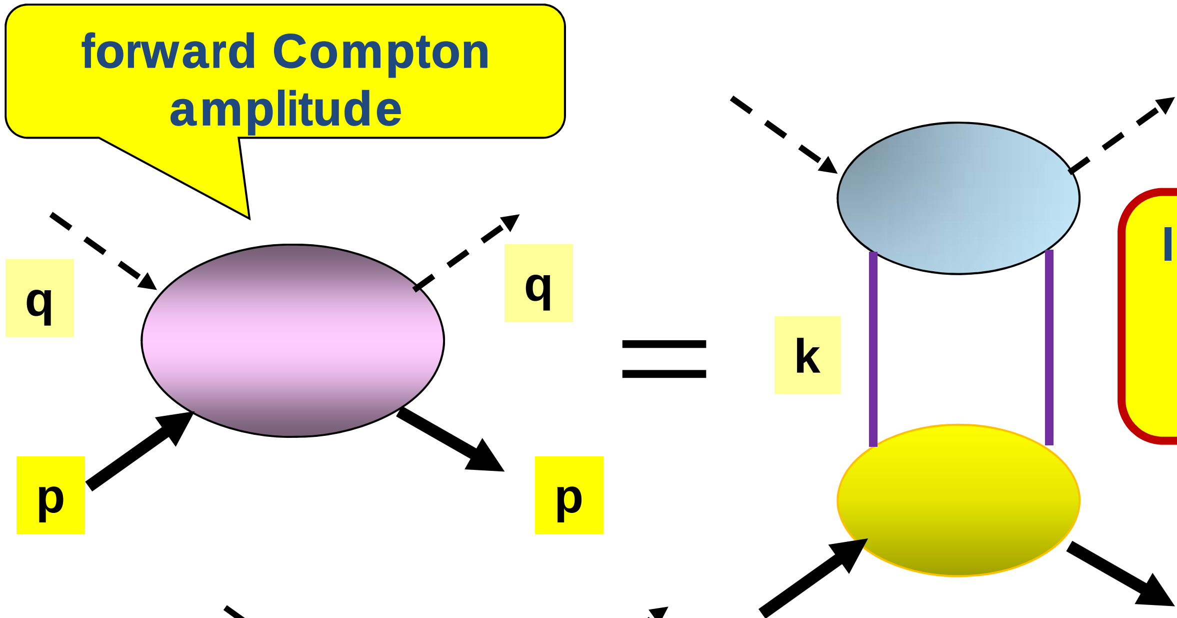
$$f^{(pert)}(x,Q^2) \otimes \delta q(x_0,\mu^2)$$



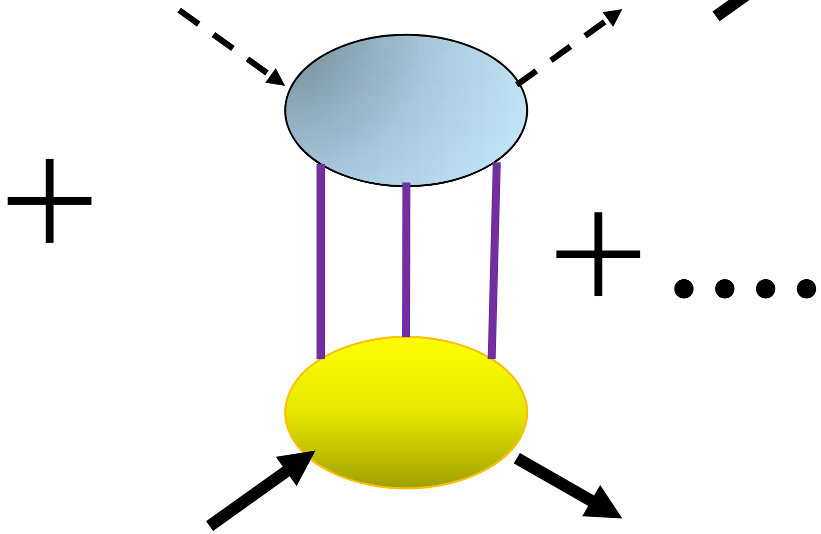
Singet F_1: x ~ 1 DGLAP,
x<<1 model-dependent description merging DGLAP and BFKL

Other DIS structure functions: non-small x: DGLAP
x<< 1 Generalizations of DGLAP to the small-x region combining x- and Q^2-evolutions (Bartels-Ermolaev-Manaenkov-Ryskin-Greco-Troyan)

DERIVATION of QCD FACTORIZATION at SMALL X



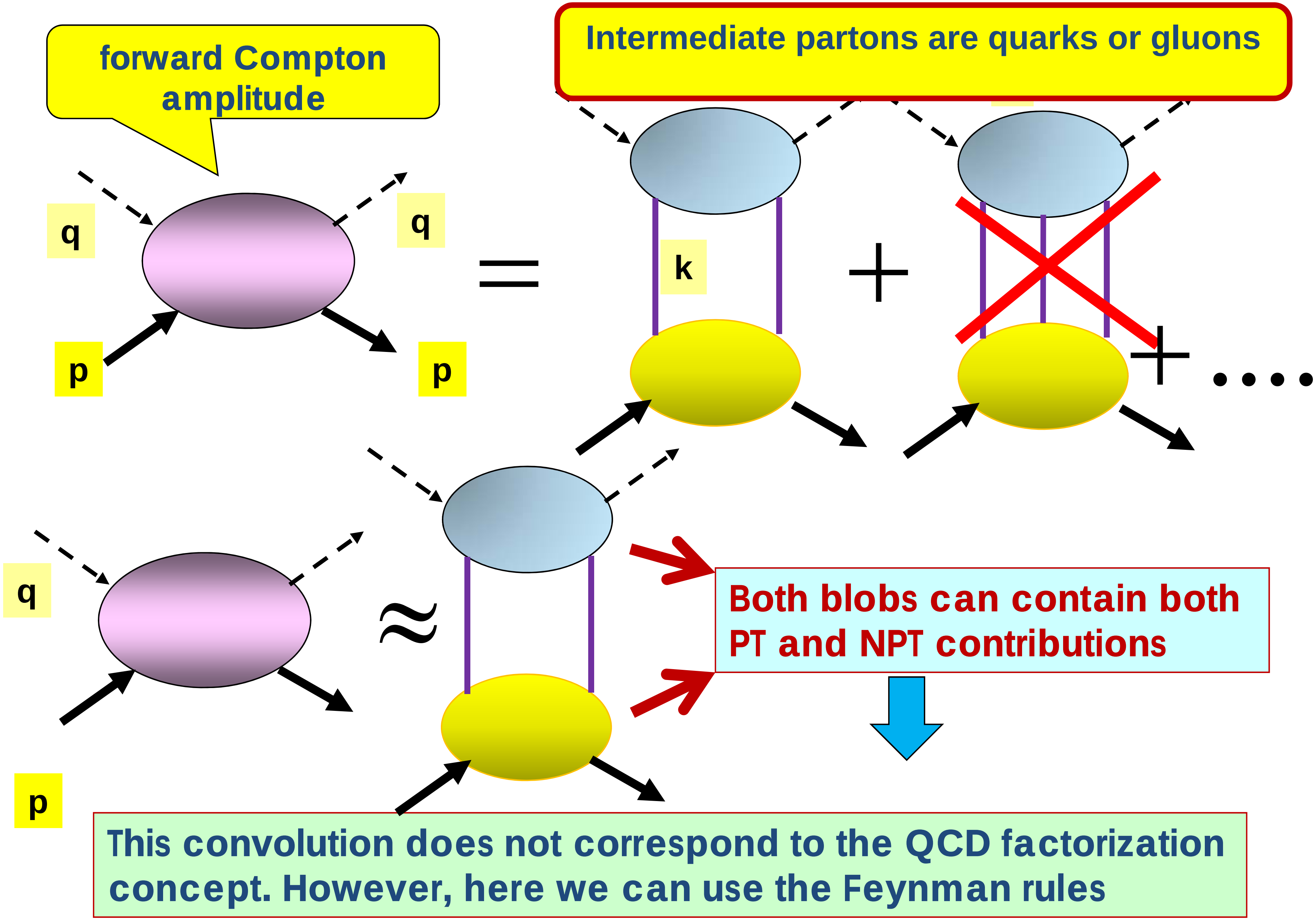
Intermediate partons in the convolutions are quarks or gluons



Next step

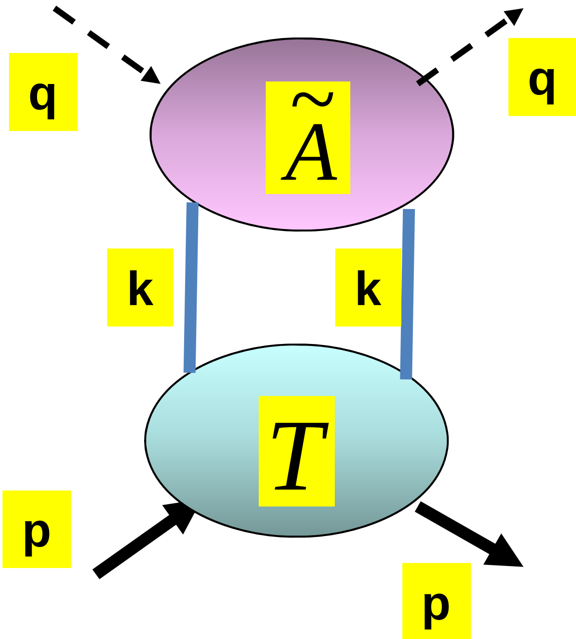
APPROXIMATION

Neglect all graphs save the first one:


forward Compton amplitude
Intermediate partons are quarks or gluons
q
q
p
p
k
q
p
Both blobs can contain both PT and NPT contributions
This convolution does not correspond to the QCD factorization concept. However, here we can use the Feynman rules

Let us focus first on the two-quark intermediate state

Applying Feynman rules we arrive at



$$A = \int \frac{d^4k}{(2\pi)^4}\, \widehat{k}\, \tilde{A}(q^2, qk, k^2)\, \widehat{k}\, \frac{1}{(k^2)^2} T(pk, k^2)$$

Integration runs over the whole phase space

We have skipped a possible dependence on variables inessential at the moment: spin, masses, etc

This representation does not correspond to QCD factorization because all blobs include both perturbative and non-perturbative contributions

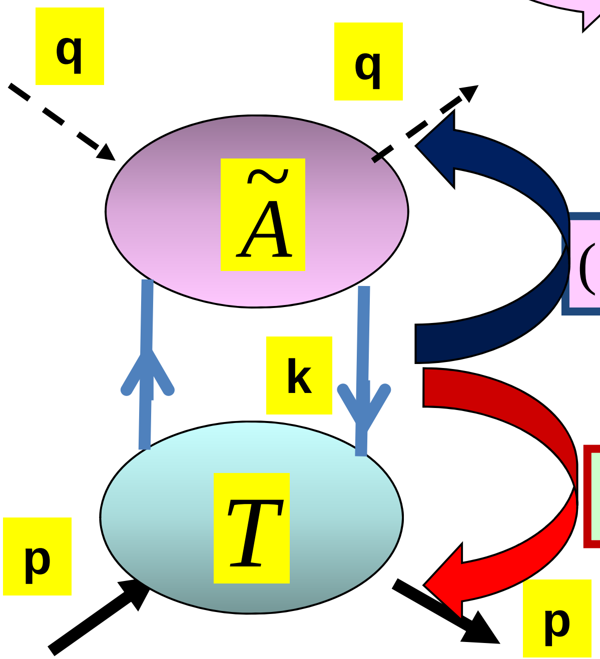
Sudakov variables:

$$\alpha, \beta, k_\perp$$

$$k = -\alpha\,(q + xp) + \beta\; p + k_\perp$$

$$k^2 = -w\alpha\beta - k_\perp^2,\;\; 2pk = -w\alpha,\;\; 2qk \approx w\beta$$

$$w = 2pq$$



$$(q+k)^2 = w\beta + q^2 + k^2;\;\; q^2 = -Q^2$$

$$(p-k)^2 = w\alpha + k^2 + p^2$$

$$A = \int \frac{d\beta}{\beta} dk_\perp^2 d\alpha\; \tilde{A}(Q^2, w\beta, k^2)\frac{1}{k^2} T(w\alpha, k^2)$$

Integration runs over the whole phase space and must yield finite result: There should be no **IR and UV divergences**

PIECE OF TERMINOLOGY: Singlet and non-singlet amplitudes:

Singlet amplitude A_S

$$\frac{1}{\pi}\,\mathrm{Im}\,A_S = F_1^S$$

vacuum quantum numbers in the t-channel

Non-singlet amplitudes A_{NS}

$$\frac{1}{\pi}\,\mathrm{Im}\,A_{NS} = F_1^{NS}, F_2, g_1^S\, g_1^{NS},\ \mathrm{etc}$$

non-vacuum quantum numbers in the t-channel

Such terminology is conventional but not altogether correct:

For instance, F_2 and flavour singlet g_1 are referred as non-singlets

Difference between singlet and non-singlet upper blobs

$$\widetilde{A_{NS}} = M_{NS}(ln \quad (w\beta/k^2), ln \quad (Q^2/k^2))$$

$$\widetilde{A_S} = \left(w/_{k^2}\right) M_S(ln \quad (w\beta/k^2), ln \quad (Q^2/k^2))$$

IR-sensitive terms
Other pert contributions are
IR-regular

comes from the first-loop graphs
with unpolarized gluons
in the t-channel

k^2 plays the role of IR cut-off
for IR-sensitive terms

singularity $k^2 = 0$ must be regulated

Integration must yield finite result: No IR and UV divergences

$$A_{NS} = \int_{-\infty}^{\infty} \frac{d\beta}{\beta} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} \frac{d\,k^2}{k^2} \widetilde{A_{NS}}(x, Q^2, \beta, k^2) T_{NS}(\alpha, k^2)$$

$$ln^n\,(w\beta/k^2),\ ln^n\,(Q^2/k^2)$$

when $k^2 \to 0$

Dealing with
IR divergences should
not involve IR cut-offs

To regulate IR
divergences, we
Impose
requirements

$$T_{NS} \sim \left(k^2\right)^{\gamma}$$

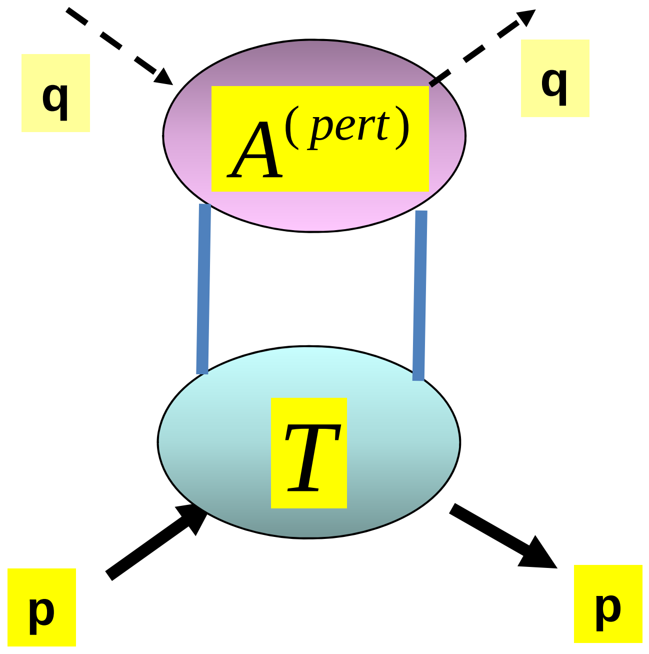
$$T_S \sim \left(k^2\right)^{1+\gamma}$$

with $\gamma > 0$

IR singularities are cut out

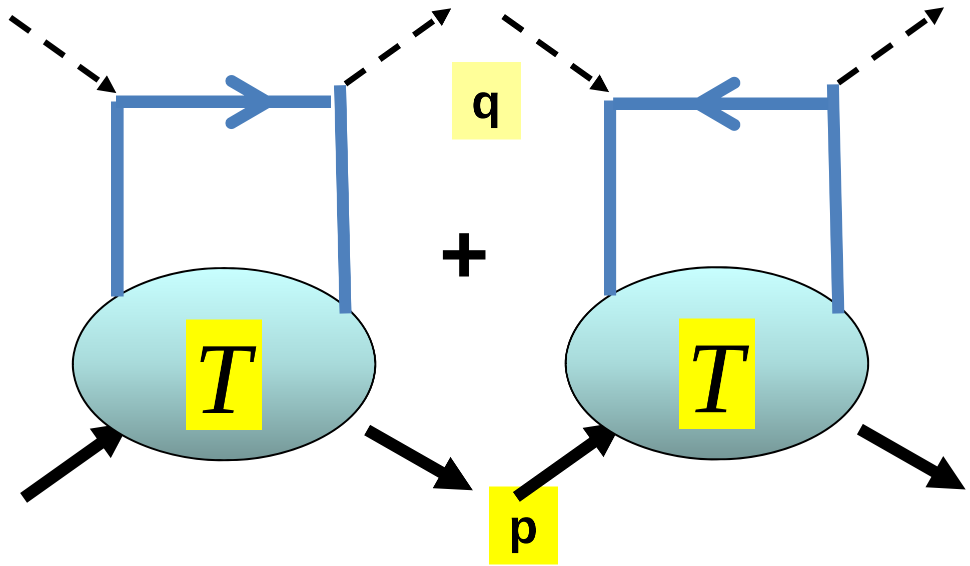
k^2 are not small, so the upper blobs are now in the Perturbative QCD domain

$$\tilde{A} = A^{(pert)}$$



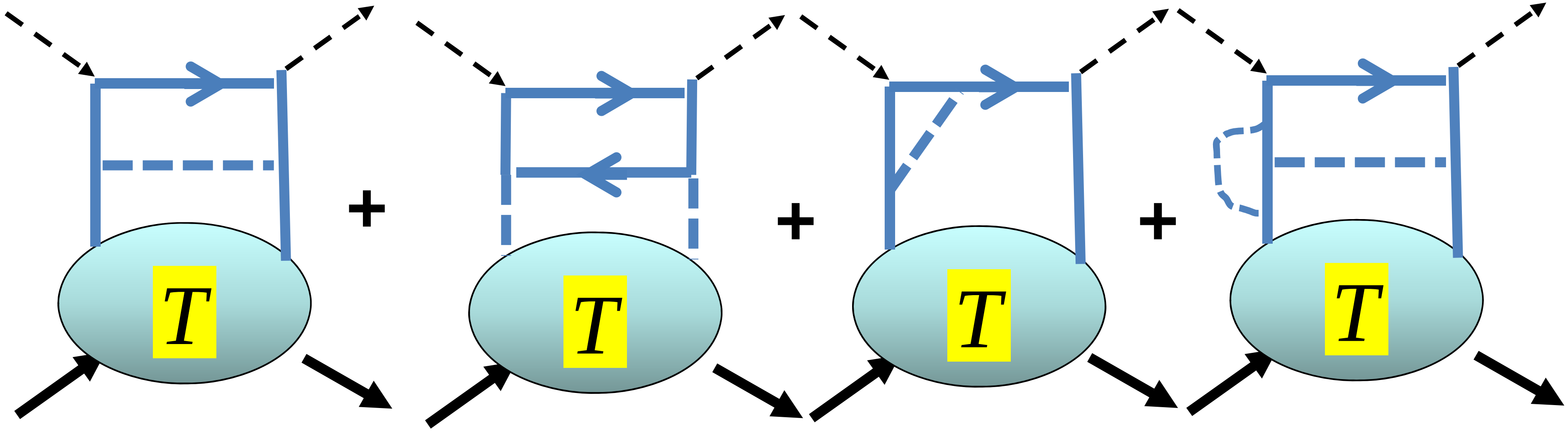
$A^{(pert)}$ can be constructed explicitly

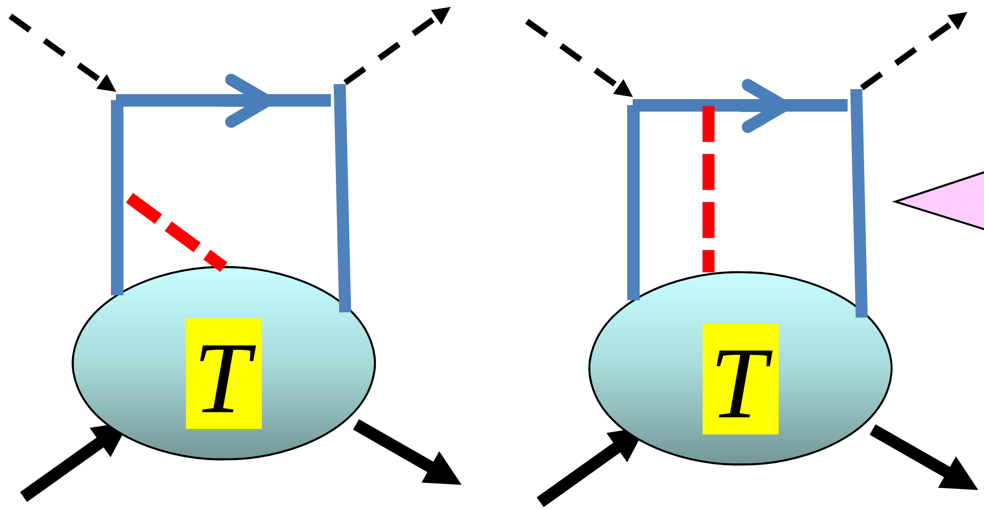
Born approximation



Radiative corrections are absent, so blob T contains non-perturbative contributions only

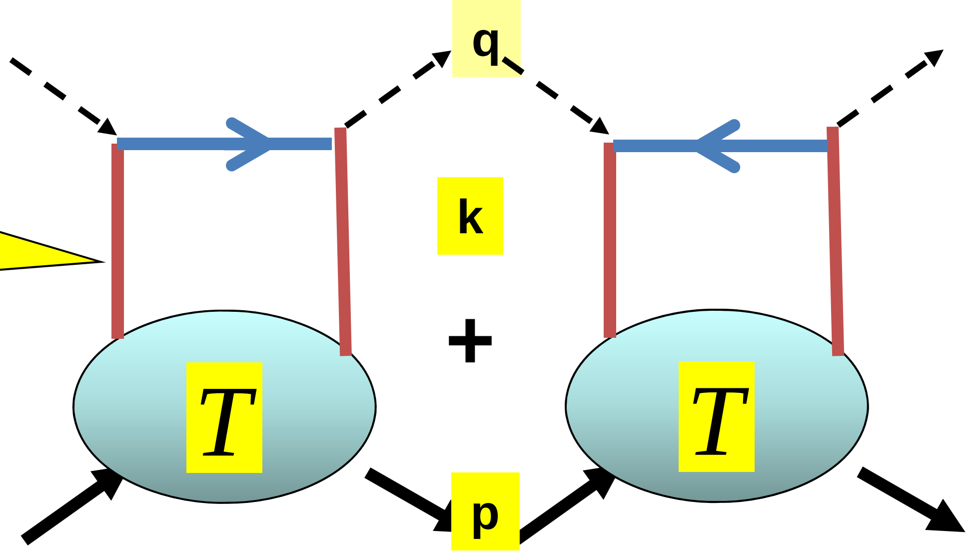
Beyond the Born approximation





Such graphs are left out
They Involve more
complicated lower blob

Remark on gauge invariance



Gauge invariance is broken when intermediate partons are off-shell

$k^2 \neq m^2$

However, typically the violation is proportional to

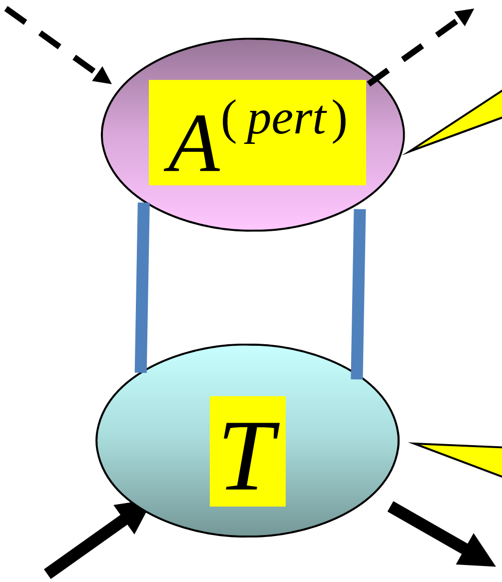
$$\frac{1}{1-x+z} - \frac{1}{1+x-z}$$

$$x = Q^2/2pq, \;\; z = (k^2 - m^2)/2pq$$

Gauge invariance is restored at small x and small z in Born approximation
Accounting for logarithmic rad corrections does not violate the invariance

This is also the applicability region for K_T - factorization: small x and accounting for logarithmic contributions

Adding more virtual partons leads to the convolution where pert and non-pert contributions are in different blobs



Perturbative blob

Non-perturbative blob

Obtained construction agrees with concept of QCD factorization but does not coincide with K_T - or collinear factorizations

$$A=\int \frac{d\beta}{\beta}\, dk_{\perp}^{2}\, \underline{d\alpha}\, A^{(pert)}(Q^2, w\beta, k^2)\frac{1}{k^2}T(w\alpha, k^2)$$

One integration in collinear factorization

Two integrations in K_T-factorization

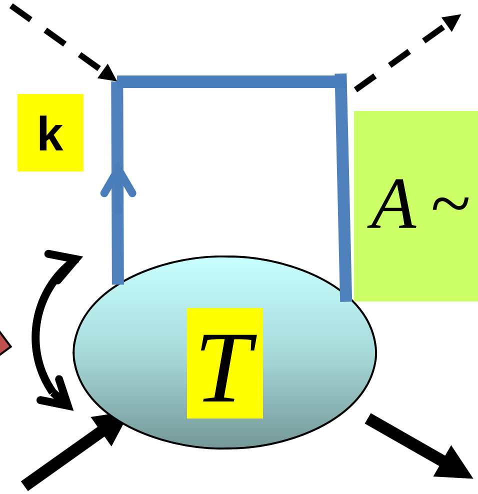
Extra integration !

New Factorization: BASIC FACTORIZATION

This construction is more general than both k_T - and collinear factorizations

Ultraviolet behavior of Born blob T at Basic Factorization

at large k



Born

$$A \sim \int \frac{1}{k^3} T_{NS}(k) d^4k = \int \frac{1}{k^3} T_{NS}(k) d\Omega dk k^3$$

$$T_{NS} \sim k^{-1-h}$$

UV stability

$$T_{NS} \sim (w\alpha)^{-1-h} \sim (s')^{-1-h}$$

$$h > 0$$

invariant energy

$$s' = (p-k)^2 \approx w\alpha$$

Accounting for radiative corrections leads to the following UV behavior:

$$T_{NS} \sim (w\alpha)^{-1-h} \sim (s')^{-1-h}$$

$$T_S \sim (w\alpha)^{-h} \sim (s')^{-h}$$

We will use this later on

Applying Optical theorem, we arrive at basic factorization for DIS structure functions:

$$f_{NS}(x,Q^2) = \int \frac{d\beta}{\beta}\, d\,k^2{}_\perp\, d\alpha\, f_{NS}{}^{(pert)}(w\beta, Q^{2\prime}\, k^2)\, \frac{1}{k^2}\, \Psi_{NS}\,(\alpha, k^2)$$

$$f_S(x,Q^2) = \int \frac{d\beta}{\beta}\, d\,k^2{}_\perp\, d\alpha\, f_S{}^{(pert)}(w\beta, Q^{2\prime}\, k^2)\, \frac{1}{k^2}\, \Psi_S\,(\alpha, k^2)$$

with totally unintegrated singlet and non-singlet parton distributions

$$\Psi_{NS} = \mathrm{Im}\, T_{NS}(w\alpha, k^2), \quad \Psi_S = \mathrm{Im}\, T_S(w\alpha, k^2)$$

$$f(x,Q^2) = \int \frac{d\beta}{\beta}\, d\,k^2_{\perp}\, d\alpha\, f^{(pert)}(w\beta, Q^{2\prime}\, k^2)\, \frac{1}{k^2}\, \Psi(\alpha, k^2)$$

Basic factorization

Transition to K_T-factorization:

integration over $\boxed{\alpha}$ should be performed without dealing with $f^{(pert)}(x, \beta, k^2)$ which is impossible to be done exactly:

$k^2 = -w\,\alpha\,\beta - k^2_{\perp}$

Approximation: $\boxed{w\,\alpha\,\beta \ll k^2_{\perp}}$

Sense: virtualities of intermediate partons are generated by their transverse momenta

$$f(x,Q^2) = \int \frac{d\beta}{\beta}\, \frac{d\,k^2_{\perp}}{k^2_{\perp}}\, f^{(pert)}(w\beta, Q^{2\prime}\, k^2_{\perp})\, \Phi(\beta, k^2_{\perp})$$

unintegrated parton distribution

K_T-factorization

$$\Phi(\beta, k^2_{\perp}) = \int_{w_0/w}^{k^2_{\perp}/w\beta} d\alpha\, \Psi(\alpha, k^2)$$

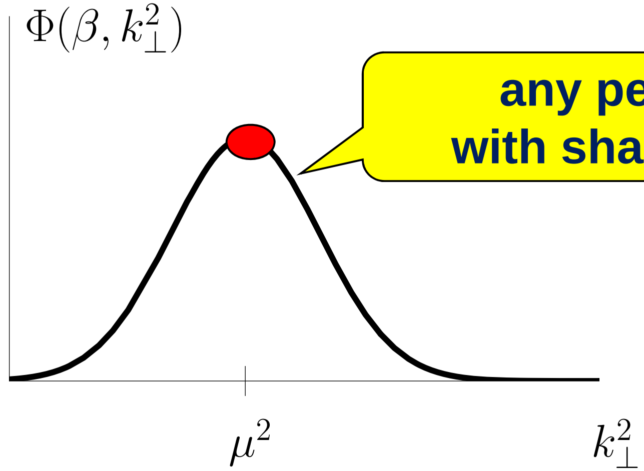
Transition from k_T - to collinear factorization

$$f(x,Q^2) = \int \frac{d\beta}{\beta}\,\frac{d\,k^2{}_\perp}{k^2{}_\perp}\, f^{(pert)}(w\beta, Q^2\, k^2{}_\perp)\,\Phi(\beta, k^2{}_\perp)$$

K_T-factorization

integration over $k_\perp^2$ should be done without integration of $f^{(pert)}(x, \beta, k^2{}_\perp)$

Assumption:



any peaked form
with sharp maximum

Integrated parton distribution

$$f(x,Q^2) = \int \frac{d\beta}{\beta}\, f^{(pert)}(w\beta, Q^2, \mu^2)\,\varphi(\beta, \mu^2)$$

$$\varphi\left(\beta,\mu^2\right) = \int_D \frac{dk^2{}_\perp}{k^2{}_\perp} k^2{}_\perp \Phi\left(\beta,k^2{}_\perp\right)$$

intrinsic factorization scale

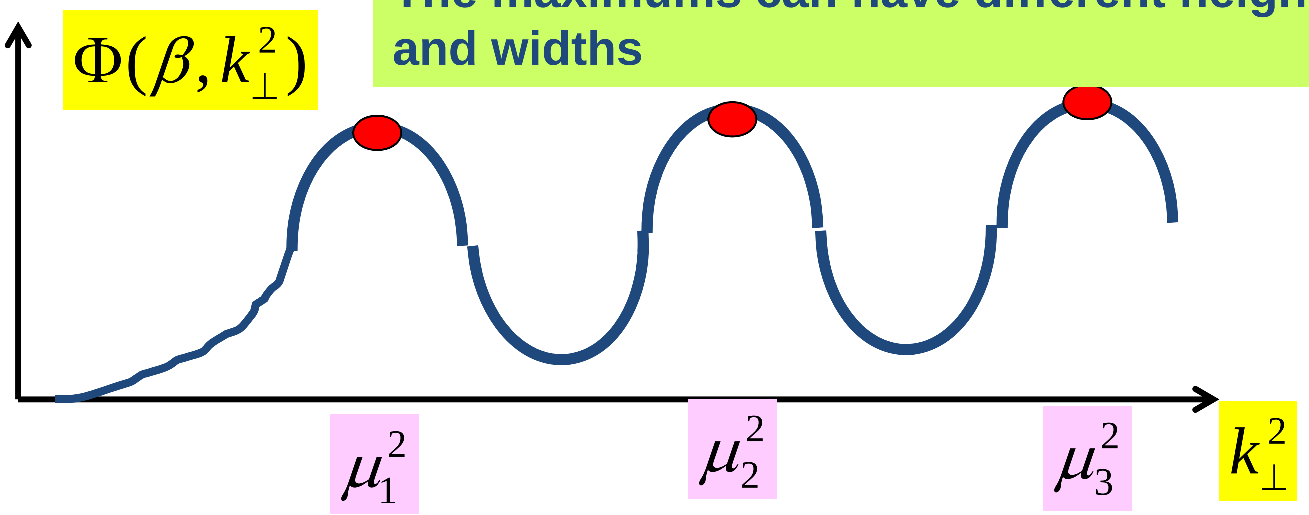
Integration runs in vicinity of maximum

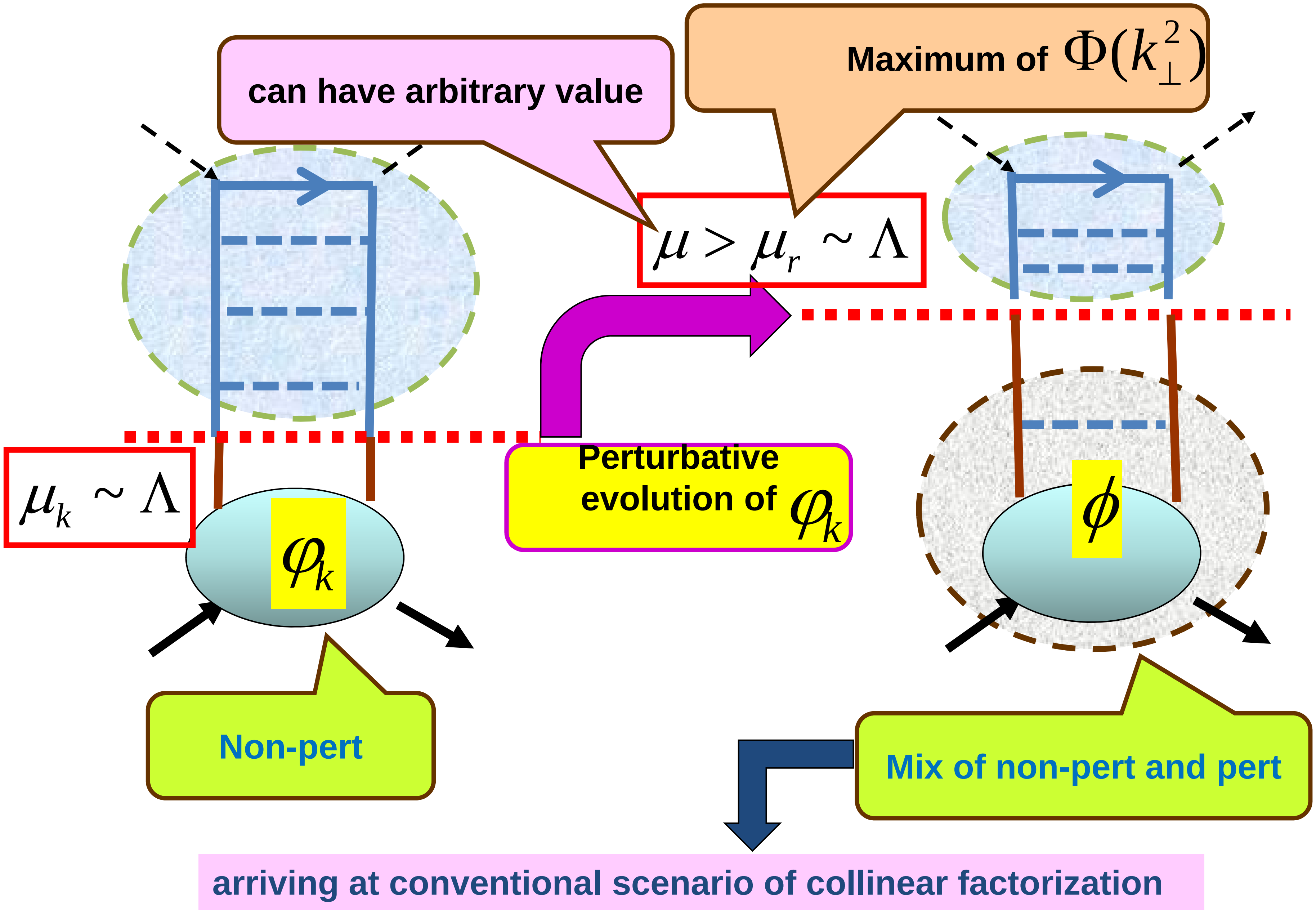
Intrinsic scale has the physical meaning: it is maximum of parton distribution in K_T –factorization and this maximum is associated with non-perturbative physics entirely: $\mu \sim \Lambda$

The sharper is the maximum, the better is accuracy of the transition from K_T to collinear factorization

More involved picture is possible: several maximums

The maximums can have different heights and widths

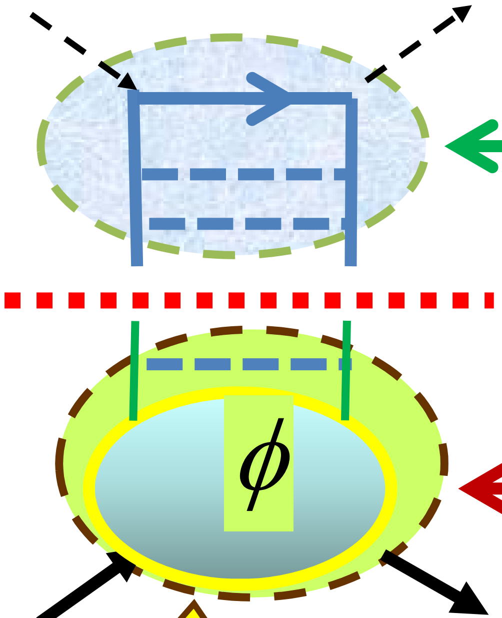




can have arbitrary value
Maximum of $\Phi(k_\perp^2)$
$\mu > \mu_r \sim \Lambda$
$\mu_k \sim \Lambda$
φ_k
Perturbative evolution of φ_k
ϕ
Non-pert
Mix of non-pert and pert
arriving at conventional scenario of collinear factorization

Conventional approach:

First, arbitrary factorization scale μ is chosen
Then, rad corr are distributed between two blobs



$k_\perp^2 > \mu^2$

often associated with staring point of Q^2 evolution

$\mu >> \mu_r \sim \Lambda$

$k_\perp^2 < \mu^2$

mix of Pert and Non-pert contributions

In conventional approach the parton distributions have much more complicated structure
and
collinear factorization is regarded as unrelated to K_T-factorization

Applications: Restrictions on fits for parton distribution

$$\Phi(x, k_\perp{}^2) = (k^2{}_\perp)^a x^h A(x, k_\perp{}^2) + (k^2{}_\perp)^b B(x, k_\perp{}^2)$$

with $0<a<h<b$

Sharp maximums in k_T

This form of the fits has recently been used by

Grinyuk-Jung-Lykasov-Lipatov-Zotov

Restrictions on DGLAP fits in collinear factorization

Generic structure of DGLAP-fits for initial parton densities:

$$\delta q(x), \delta q(x) \sim \underbrace{N}_{\text{normalization}} \underbrace{x^{-a}}_{\text{singular factor}} \underbrace{(1-x)^b (1+cx^d)}_{\text{regular term}} \qquad N, a, b, c, d > 0$$

Fits are introduced from purely phenomenological considerations to explain experimental data

Several years ago we proved that the role of singular factors is to mimic resummation of logarithms of x. These factors should be dropped when resummation is accounted for.

Now we show that hey also contradict integrability of Compton amplitudes

UV-stability of Compton amplitude
In basic factorization

$$T_{NS} \sim \alpha^{-1-h} \qquad T_S \sim \alpha^{-h}$$

K_T and collinear factorizations

$$T_{S,NS}(\alpha) \Rightarrow \Phi_{S,NS}(\beta)$$

and

$$\alpha \Rightarrow k_\perp^2 / w\beta$$

$$\Phi_{NS} \sim \beta^{h} \qquad \Phi_S \sim \beta^{-1+h}$$

$$\delta q_{NS} \sim x^{-a}$$

excludes

$$\delta q_S \sim x^{-a} \quad \text{with } a \geq 1$$

In more detail

singlet F_1^S

NO FACTORS

x^{-a} with $a \geq 1$

All spin-dependent str functions $g_1^S, g_1^{NS}, \ldots$

unpolarized str functions F_2, F_1^{NS}

NO FACTORS

x^{-a} with $a > 0$

Restriction for non-singlets is stricter

The same parton distributions contribute to both F_1^S and F_2

Universal restriction NO FACTORS x^{-a} with $a > 0$ for for all DIS structure functions because of integrability of the convolutions

Simplification of the fit down to

$$\delta q(x), \delta q(x) \sim N\,(1-x)^b$$

CONCLUSION

Forward Compton amplitude and DIS structure functions can approximately be represented as convolutions of two blobs, where PT and NPT contributions are located in different blobs.
We call it Basic QCD Factorization

We obtain both K_T – and Collinear Factorizations with step-by-step reductions of Basic QCD Factorization Transition to K_T – factorization is based on purely mathematical restriction

In contrast, transition to Collinear Factorization presumes peaked K_T – dependence of unintegrated parton distributions.

New (intrinsic) scale of Collinear Factorization corresponds to maximum(s) of parton distribution in K_T -Factorization and is originated by non-perturbative physics, so the parton distributions at this scale are totally non-pertrurbative. This form and standard form of Collinear Factorization are related by perturbartve evolution

Application to fits for parton distributions

Fits in K_T –factorization should include two terms, each with factor $(k^2{}_\perp)^a$, with different exponents These factors are multiplied by functions with peaked dependence on $k^2{}_\perp$

Fits used in DGLAP in collinear factorization should not involve singular factors x^{-a}

Singular factors in DGLAP-fits are intended to mimic total resummation of logarithms of x. Their impact makes possible to extend DGLAP to small x and they can be dropped when the logarithms are resummed

In addition, we have shown that these factors are excluded by mathematic reasons