

Baryon as Dyonic Instanton

Krikun, A.G. 2012

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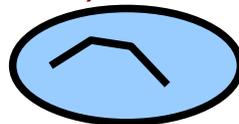
S-Petersburg, October 4

Where the baryon mass come from?

1. Mass is defined by the conformal anomaly
2. Mass is defined by the value of the Skyrme term
3. Mass is defined by the quark condensate (Ioffe,91)

$$m_N^3 = -8\pi^2 \langle 0 | \bar{q}q | 0 \rangle .$$

Skyrmion with $B > 2$ has the torus topology (Kopeliovich-Stern, Manton, Verbaarschot 87)



There are nontrivial phase transitions to the Half Skyrmion state at the nonvanishing density (Rho et.al ,Sonnenchein et.al)

The mass of the nucleon in the matter reduces considerably upon the restoration of the chiral symmetry

Is it possible to combine these results and observations in the generalized Skyrmion picture?

To what extent the nucleon mass is related to the chiral condensate?

New mechanism for the instanton size stabilization.

YM + real scalar in D=5 (N=1 SYM), but SUSY is not too important. (Lambert-Tong 99)

$$\rho = \sqrt{\frac{Q_e}{4\pi^2 v}}$$

Solution to the equation of motion with two charges

$$F_{\mu\nu} = *F_{\mu\nu}, \quad D_\mu \phi = E_\mu, \quad D_0 \phi = 0,$$

$$M = \frac{4\pi^2}{g^2} |I| + |v Q_e|,$$

The Hamiltonian reads as

$$H = \frac{1}{2} \text{Tr} \int d^4x \left\{ (E_\mu - \mathcal{D}_\mu \phi)^2 + \frac{1}{4} (F_{\mu\nu} - {}^*F_{\mu\nu})^2 + (\mathcal{D}_0 \phi)^2 + \frac{1}{2} F_{\mu\nu} {}^*F_{\mu\nu} + 2E_\mu \mathcal{D}_\mu \phi \right\}$$

In the singular gauge the solution is

$$A_\mu = \frac{2}{g} \frac{\rho^2}{x^2(x^2 + \rho^2)} \eta_{\mu\nu}^a x_\nu \frac{\sigma^a}{2} \quad ; \quad \phi = v \frac{x^2}{x^2 + \rho^2} \frac{\sigma^3}{2} ,$$

The electric charge is defined via scalar current

$$E_\mu = \mathcal{D}_\mu \phi = \frac{2v\rho^2 x_\nu}{(x^2 + \rho^2)^2} \eta_{\nu\lambda}^3 \eta_{\mu\lambda}^a \frac{\sigma^a}{2} . \quad q = \int d^4S_\mu \frac{1}{v} \text{Tr}(\phi E_\mu) = 4\pi^2 \rho^2 v$$

It does not vanish at the dyonic instanton solution!

The dyonic instanton has the transparent brane interpretation

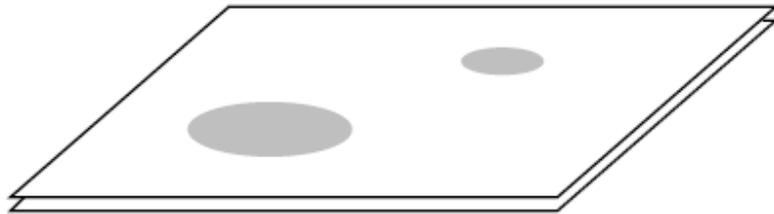


Figure 1: D-brane representation of the Yang-Mills instantons. The D0-branes (resolved by condensation of D0-D4 strings) are sitting inside the parallel coincident D4-branes.

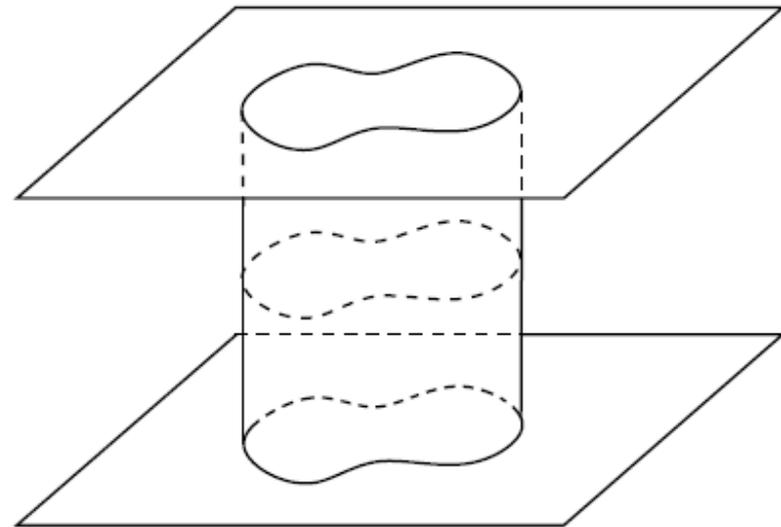


Figure 2: A dyonic instanton. The D2-brane supertube is suspended between two parallel D4-branes.

$$D4 + D0 + F1 \rightarrow D4 + D2(\text{cylinder}) + \text{charge densities}$$

The Lagrangian of D2 worldvolume theory

$$L = -\sqrt{R^2(1 - E^2) + B^2},$$

Canonical momentum and Hamiltonian

$$\Pi = \frac{\partial L}{\partial E} = \frac{R^2 E}{\sqrt{R^2(1 - E^2) + B^2}} \quad \mathcal{H} = R^{-1} \sqrt{(\Pi^2 + R^2)(B^2 + R^2)}$$

Two charges (electric and topological)

$$Q_F = \frac{1}{2\pi} \oint d\phi \Pi, \quad Q_0 = \frac{1}{2\pi} \oint d\phi B$$

If the topological charge is two or larger the solution looks as the closed monopole string with distributed topological charge

The tension of the tube is defined as

$$T = \frac{1}{2\pi} \oint d\phi \mathcal{H},$$

Can be written as the sum of two terms without the D2 charge

$$T = |Q_F| + |Q_0|$$

The dyonic instanton carries the angular momentum (Townsend,2001)

$$L = Q_F Q_0$$

which coincides with the cross-section of the tube

$$L = \oint ds \left(x_3 \frac{\partial x_4}{\partial s} - x_4 \frac{\partial x_3}{\partial s} \right)$$

It is responsible for the stabilization !

Skyrmion = instanton in D=5 gauge theory (Son,Stephanov 04)

The gauge theory involves the flavor as gauge group and is defined on the worldvolume of the flavor branes

$$S = \int d^3x dt dz \left\{ \frac{1}{z} \left(-\frac{1}{4g_5^2} \right) (F_L^2 + F_R^2) + \frac{\Lambda^2}{z^3} (DX)^2 + \frac{\Lambda^2}{z^5} 3|X|^2 \right\}$$

The simplest hard-wall model. It is D=5 theory with two gauge groups (left + right) and bifundamental scalar. The fifth coordinate z represents the RG scale in our 3+1 world. It plays the role of «time» for the instanton solution.

The Gauss Law in D=5 = anomaly in D=4

Conventional Stabilization of the Skymion size

1. Add Skyrme term.
2. Possible stabilization via the Chern-Simons term
3. Dynamical realization of the Atiyah-Manton picture ('89).

$$g(\mathbf{x}) = \mathcal{P} \exp \left(i \int_{-\infty}^{+\infty} dx^4 A_4(\mathbf{x}, x^4) \right) .$$

The instanton holonomy along the time direction yields the Skymion field with very high accuracy. Dynamically: the instanton gets captured by some «wall» in the fifth coordinate and width of the wall fixes the instanton (Skymion) radius (Eto, Nitta, Sakai, Tong 05). Additional field with the nontrivial z dependence is required.

Conjecture: Baryon = Dyonic instanton in the flavor gauge group (Krikun ,A..G. '12)= Skyrmion which takes into account the meson tower and chiral condensate

A few generalizations of the Lambert-Tong solution are required

1. Two gauge groups
2. The bifundamental scalar
3. The curved geometry

Has some similarity with the Atiyah-Manton picture since the solution for the bifundamental scalar is nontrivial

$$Q_B = \frac{1}{32\pi^2} \int d^3x \int_{\epsilon}^{z_m} dz \left\langle F_L \tilde{F}_L - F_R \tilde{F}_R \right\rangle ,$$

Consider Witten's cylindric ansatz for the instanton, '77

$$A_j^a = \frac{1 + \varphi_2(r, z)}{r} \epsilon_{jak} \frac{x_k}{r} + \frac{\varphi_1(r, z)}{r} \left(\delta_{ja} - \frac{x_j x_a}{r^2} \right) + A_r(r, z) \frac{x_j x_a}{r^2}$$
$$A_z^a = A_2(r, z) \frac{x_a}{r} \quad r^2 = x_1^2 + x_2^2 + x_3^2$$

and for bifundamental

$$X = \chi_0(r, z) \frac{1}{2} + i\chi_1(r, z) \frac{\tau^a x^a}{r}$$

The boundary conditions (more general later)

$$A_i^L(x, z) = -A_i^R(-x, z), \quad L_z(x, z) = R_z(-x, z)$$

The Lagrangian reduces to the abelian gauge field with two complex scalars in 2+1 theory (1+1 in the static case)

$$\varphi = \varphi_1 + i\varphi_2 \equiv \varphi e^{i\alpha},$$

$$\chi = \chi_0 + i\chi_1 \equiv \chi e^{i\beta},$$

with the action

$$S = 4\pi \int dt \int dr dz \left\{ -\frac{1}{2g_5^2} \left[\frac{2}{z} |D_a \varphi|^2 + \frac{1}{2} (F_{ab})^2 + \frac{1}{r^4} (1 - |\varphi|^2)^2 \right] \right. \\ \left. - \frac{\Lambda^2 r^2}{z^3} \left[\frac{1}{2} |D_a \chi|^2 + \frac{1}{2r^2} (|\chi|^2 + |\varphi|^2 - |\varphi + \chi|^2) \right] \right. \\ \left. + \frac{\Lambda^2 r^2}{z^5} \frac{3}{2} |\chi|^2 \right\},$$

$$Q_B = \frac{1}{2\pi} \int dr \int dz \epsilon_{ab} \left(\partial_a (-i\varphi D_b \bar{\varphi} - h.c.) + F_{ab} \right)$$

The baryonic charge

Introduce two phases

$$\frac{\Lambda^2}{2z^3} (|\chi|^2 + |\varphi|^2 - |\varphi + \chi|^2) = \frac{\Lambda^2}{z^3} \chi^2 \varphi^2 \cos(\alpha - \beta)^2$$

$$\gamma = \alpha - \beta - \frac{\pi}{2}. \quad \gamma|_{vac} = \pi N, \quad N \in \mathbb{Z}.$$

The chiral condensate enters the game via the boundary condition

$$\varphi(r, z)|_{z=0} = 1, \quad \chi(r, z)|_{z=0} \sim \sigma z^3, \quad A_r(r, z)|_{z=0} \sim z^2$$

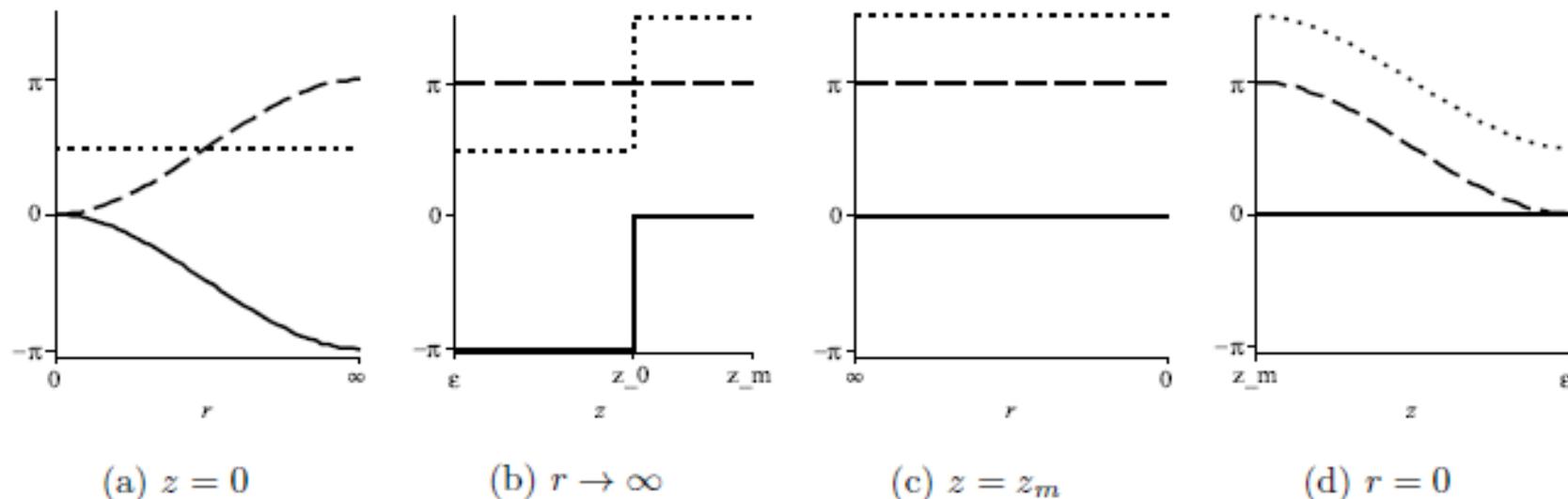


FIG. 2: The qualitative boundary behavior of the phases α (dotted), β (dashed) and γ (solid). (Horizontal axis of (c) and (d) reversed.)

Energy of the Dyonic Skyrmion

$$\begin{aligned}
 E = \frac{N_c}{6\pi} \int dr dz \left\{ & -\frac{2}{z}(\partial_z \varphi)^2 - \frac{2}{z}(\partial_r \varphi)^2 \right. \\
 & -\frac{1}{z}\varphi^2 \left[\cos(\theta)^2(A_z - \partial_z \alpha)^2 + \sin(\theta)^2(A_z - \partial_z \alpha + \partial_z \omega)^2 + \cos(\omega)^2(\partial_z \theta)^2 \right. \\
 & \quad \left. + (\sin(\omega)\partial_z \theta + V_z)^2 + (2A_z - 2\partial_z \alpha + \partial_z \omega) \sin(2\theta)V_z \cos(\omega) \right] \\
 & -\frac{1}{z}\varphi^2 \left[\cos(\theta)^2(A_r - \partial_r \alpha)^2 + \sin(\theta)^2(A_r - \partial_r \alpha + \partial_r \omega)^2 + \cos(\omega)^2(\partial_r \theta)^2 \right. \\
 & \quad \left. + (\sin(\omega)\partial_r \theta + V_r)^2 + (2A_r - 2\partial_r \alpha + \partial_r \omega) \sin(2\theta)V_r \cos(\omega) \right] \\
 & -\frac{r^2}{2z}(\partial_z A_r - \partial_r A_z)^2 - \frac{r^2}{2z}(\partial_z V_r - \partial_r V_z)^2 - \frac{r^2}{2z}(\partial_z a_r - \partial_r a_z)^2 \\
 & -\frac{1}{r^2 z}(1 - \varphi^2)^2 - \frac{1}{r^2 z}\varphi^4 \sin(2\theta)^2 \cos(\omega)^2 \\
 & -\frac{3r^2}{z^3}(\partial_z \chi)^2 - \frac{3r^2}{z^3}\chi^2(\partial_z \gamma - A_z)^2 - \frac{3r^2}{z^3}\chi^2(a_z)^2 \\
 & -\frac{3r^2}{z^3}(\partial_r \chi)^2 - \frac{3r^2}{z^3}\chi^2(\partial_r \gamma - A_r)^2 - \frac{3r^2}{z^3}\chi^2(a_r)^2 \\
 & -\frac{6}{z^3}\chi^2\varphi^2 \left[\cos(\theta)^2 \cos(\gamma - \alpha)^2 + \sin(\theta)^2 \sin(\gamma - \alpha + \omega)^2 \right] \\
 & \left. + \frac{9r^2}{z^5}\chi^2 \right\}
 \end{aligned}$$

Features of the solution

1. It does not fall on the boundary wall
2. It is stabilized by the second charge, related to the axial charge
3. The essential contribution to the mass is due to the chiral condensate.
4. The main difference with the standard Skyrmion; Infinite number of mesons is taken into account. The chiral condensate is taken into account

The key result

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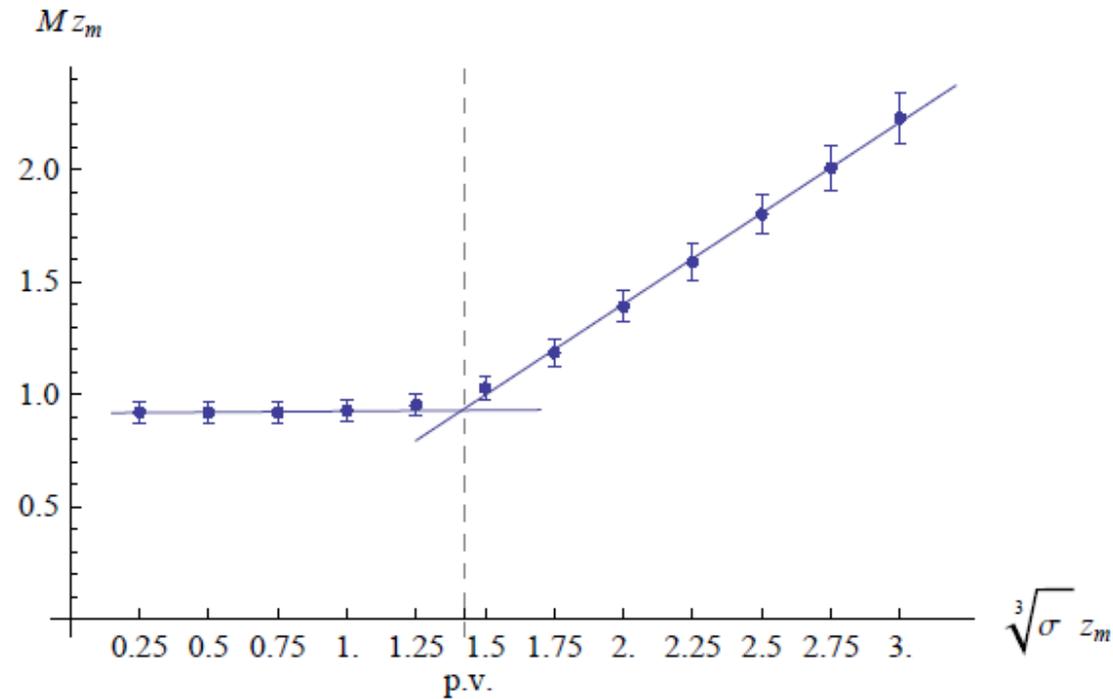


FIG. 2: The dependence of the baryon mass on the chiral condensate. The lines are constructed by the least squares fit. The fit is well inside the 5% precision error bars of the energy values. The dashed line represents the physical values (p.v.) of the parameters, which provide the fit of the hard wall model to the real observables.

More general boundary conditions (no V-A symmetry)
(Kopnin, Krikun A.G.'13)

$$\eta_1 = \phi \cos(\theta) \cos(\alpha)$$

$$\eta_2 = \phi \cos(\theta) \sin(\alpha)$$

$$\xi_1 = \phi \sin(\theta) \cos(\beta)$$

$$\xi_2 = \phi \sin(\theta) \sin(\beta)$$

The potential energy from the gauge part

$$\frac{1}{r^4} \left[(\phi^2 - 1)^2 + \phi^4 \sin(2\theta)^2 \cos(\alpha - \beta)^2 \right]$$

and the scalar part

$$|\chi_1 \eta_1 + \chi_2 \eta_2|^2 = \chi^2 \varphi^2 \cos(\theta)^2 \cos(\gamma - \alpha)^2$$

$$|\chi_1 \xi_2 - \chi_2 \xi_1|^2 = \chi^2 \varphi^2 \sin(\theta)^2 \sin(\gamma - \beta)^2$$

The baryon (topological) charge and the axial isoscalar charge are total derivatives and are similar to two charges of the dyonic instanton solution.

$$Q_B \sim F_L^{5i} \vec{F}_L^i - F_R^{5i} \vec{F}_R^i = \frac{1}{r^2} \partial_r \left[(\varphi^2 - 1) (A_z - \partial_z \alpha) + \varphi^2 (V_z \sin(2\theta) \cos(\omega) - \sin(\theta)^2 \partial_z \omega) \right] - \frac{1}{r^2} \partial_z \left[(\varphi^2 - 1) (A_r - \partial_r \alpha) + \varphi^2 (V_r \sin(2\theta) \cos(\omega) - \sin(\theta)^2 \partial_r \omega) \right]$$

$$Q_5 \sim F_L^{5i} \vec{F}_L^i + F_R^{5i} \vec{F}_R^i = \frac{1}{r^2} \partial_r \left[(\varphi^2 - 1) V_z + \varphi^2 \cos(\omega) \left((A_z - \partial_z \alpha - \frac{1}{2} \partial_z \omega) \sin(2\theta) - \partial_z \theta \right) \right] - \frac{1}{r^2} \partial_z \left[(\varphi^2 - 1) V_r + \varphi^2 \cos(\omega) \left((A_r - \partial_r \alpha - \frac{1}{2} \partial_r \omega) \sin(2\theta) - \partial_r \theta \right) \right]$$

Isvector charges are not total derivatives

$$Q_b^a \sim \hat{F}_L \tilde{F}_L - \hat{F}_R \tilde{F}_R = \frac{x^a}{r} \frac{2}{r^2} \left[(\dot{a}_1 - a'_2) (1 - \varphi^2) \right]$$
$$Q_5^a \sim \hat{F}_L \tilde{F}_L - \hat{F}_R \tilde{F}_R = -\frac{x^a}{r} \frac{2}{r^2} \left[(\dot{a}_1 - a'_2) \varphi^2 \sin(2\theta) \cos(\omega) \right]$$

With the proper boundary conditions

$$Q_B \sim \int F_L^{5i} \vec{F}_L^i - F_R^{5i} \vec{F}_R^i = \int_0^\infty dr (\partial_r \omega)|_{z=z_{IR}} = - \int_0^\infty dr (\partial_r \gamma)|_{z=z_{IR}} = \pi$$

$$Q_5 \sim \int F_L^{5i} \vec{F}_L^i - F_R^{5i} \vec{F}_R^i = 0$$

Compare with the conventional Skyrmion ansatz

$$U(r) = 1 \cos(\theta(r)) + \frac{x^a \tau^2}{r} \sin(\theta(r))$$

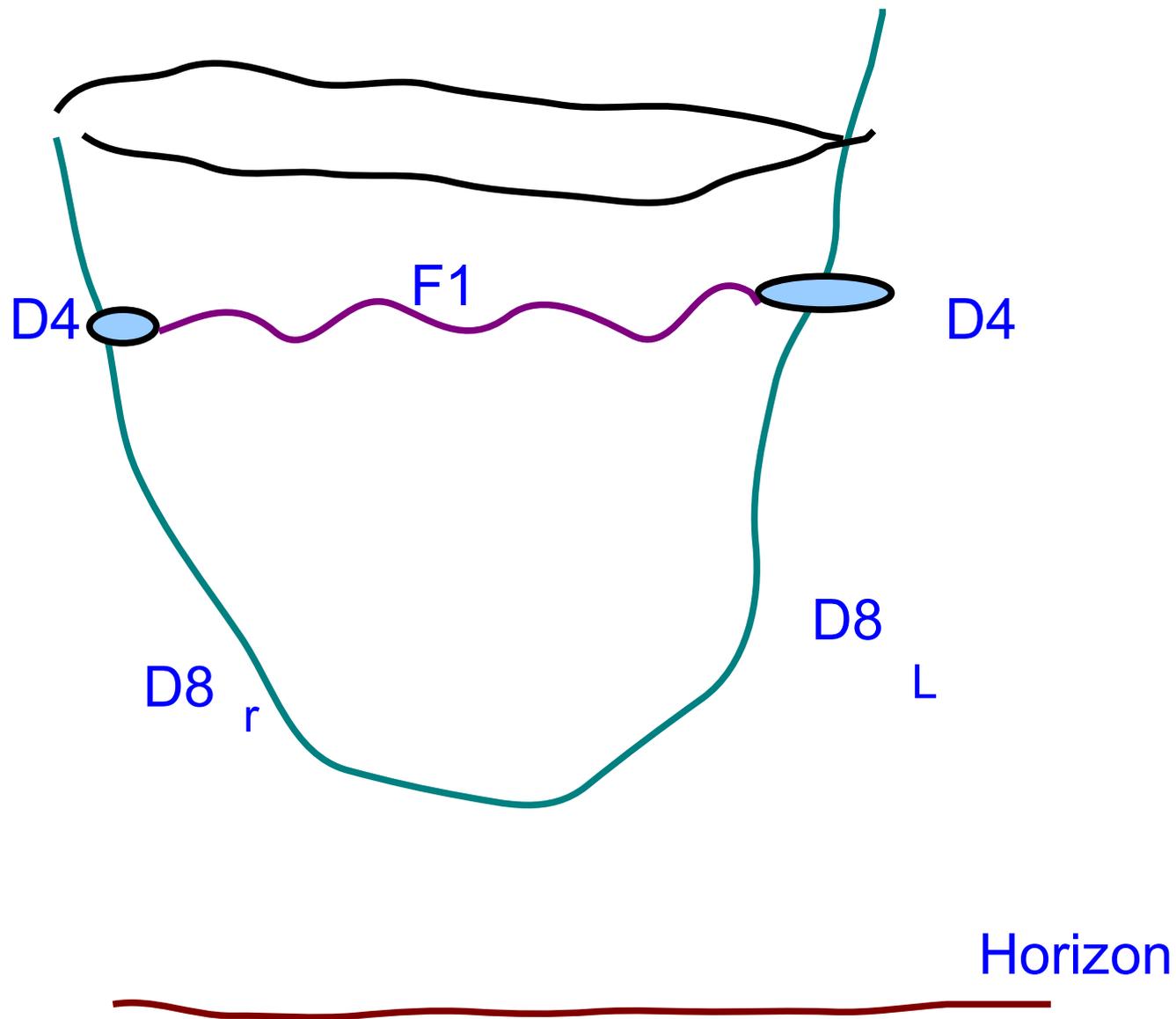
$$Q_s = \frac{\theta(0) - \theta(\infty)}{\pi} = 1$$

In our dyonic instanton solution we have

$$X(r, z) = |X(z)| \left(1 \cos(\gamma(r, 0)) + \frac{x^a \tau^2}{r} \sin(\gamma(r, 0)) \right)$$

$$Q_s = \frac{\gamma(0, 0) - \gamma(\infty, 0)}{\pi} = 1$$

Brane picture in the Witten-Sakai-Sugimoto cigar geometry



Conclusion

New mechanism for the Skyrmion size stabilization. Chiral condensate is involved

Energy of the dyonic Skyrmion solution is too large extent (numerically) due to the chiral condensate (Ioffe-like picture)

There should be very subtle mechanism due to competition between the conformal symmetry breaking and chiral symmetry breaking

$$M_B \propto \langle B | T_{\mu\mu} | B \rangle$$

$$T_{\mu\mu} = \beta \text{Tr} G^2 + m \bar{q} q$$

Torus - like configurations are very natural for $B > 2$.
(fits with previous analysis)

Possible nontrivial contribution to the angular momentum due to two charges similar to the dyonic instanton case