Neutron star models from $f(R)$ gravity

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The current accelerated expansion of the universe has been confirmed by several independent observations. Standard candles and distance indicators point out an accelerated expansion which cannot be obtained by ordinary perfect fluid matter as source for the cosmological Friedmann equations (*Perlmutter et al.(1999), Riess et al.(1998)*).
In particular, the discrepancy between the amount of luminous matter revealed from observations and the critical density needed to obtain a spatially flat universe could be solved if one assumes the existence of a non-standard cosmic fluid with negative pressure, which is not clustered in large scale structure. In the simplest scenario, this *dark energy*, can be addressed as the Einstein Cosmological Constant and would contribute about 70% to the global energy budget of the universe.

From an observational viewpoint, this model has the feature to be in agreement with data coming from observations. It could be assumed as the first step towards a new standard cosmological model and it is indicated as Concordance Lambda Cold Dark Matter (ΛCDM) Model. Despite of the agreement with observations, the ΛCDM model presents incongruences from a theoretical viewpoint. If the cosmological constant constitutes the "vacuum state" of the gravitational field, we have to explain the 120 orders of magnitude between its observed value at cosmological level and the one predicted by any quantum gravity (*Weinberg(1989)*). A very straightforward approach is to look for explanations for dark matter and dark energy within the realm of known physics. On the other hand, an alternative is that General Relativity is not capable of describing the universe at scales larger than Solar System, and dark components (energy + matter) could be the observable effect of such an inadequacy. Assuming this point of view, one can propose alternative theories of gravity extending the Einstein theory (in this sense one deals with modified gravity), keeping its positive results, without requiring dark components, up to now not detected at experimental level. In this perspective, it can be shown that the accelerated expansion can be obtained without using new fundamental ingredients but enlarging the gravitational sector (see for example *Capozziello(2002), Nojiri and Odintsov (2003)*).

In particular, it has been recently shown that such theories give models able to reproduce the Hubble diagram derived from SNela surveys (*Capozziello and Faraoni(2010)*).
However, also this approach needs new signatures or some *experimentum crucis* in order to be accepted or refuted. In particular, exotic astrophysical structures, which cannot be addressed by standard gravity, could constitute a powerful tool to address this problem. In particular, strong field regimes of relativistic astrophysical objects could discriminate between General Relativity and its possible extensions.
The study of relativistic stars in modified gravity could have very interesting consequences to address this issue. In fact, new theoretical stellar structures emerge and they could have very important observational consequences constituting the signature for the Extended Gravity (see e.g. *Capozziello et al.(2012)*). Furthermore, strong gravitational regimes could be considered if one assume General Relativity as the weak field limit of some more complicated effective gravitational theory (*Psaltis(2008)*). In particular, considering the simplest extension of General Relativity, namely the $f(R)$ gravity, some models can be rejected because do not allow the existence of stable star configurations (*Bamba et al.(2008), Nojiri and Odintsov(2009)*). On the other hand, stability can be achieved in certain cases due to the so called *Chameleon Mechanism* (*Tsujikawa et al.(2009)*).

We investigate the R^2 model with logarithmic $[f(R) = R + \alpha R^2(1 + \beta ln(R/\mu^2)]$ and cubic $[f(R) = R + \alpha R^2(1 + \gamma R)]$ corrections. In particular, we consider the FPS and SLy equations of state and a case of piecewise EoS for neutron stars with quark cores. One of the results is that, if cubic term, at some densities, is comparable with the quadratic one, stable star configurations exist at high central densities. The minimal radius of such stars is close to 9 km for maximal mass $\sim 1.9 M_{\odot}$ (SLy equation) or to 8.5 km for mass $\sim 1.7 M_{\odot}$ (FPS equation). Clearly, such objects cannot be achieved in the context of General Relativity so their possible observational evidences could constitute a powerful probe for modified gravity.

Modified TOV equations in $f(R)$ gravity

Let us start from the action for $f(R)$ gravity:

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_{\mathrm{matter}} \quad , \tag{1}$$

where g is determinant of the metric $g_{\mu\nu}$ and S_{matter} is the action of the standard perfect fluid matter. The field equations for metric $g_{\mu\nu}$ can be obtained by varying with respect to $g_{\mu\nu}$. It is convenient to write function $f(R)$ as

$$f(R) = R + \alpha h(R), \tag{2}$$

where $h(R)$ is an arbitrary function.
In this notation, the field equations are

$$(1 + \alpha h_R) G_{\mu\nu} - \frac{1}{2}\alpha(h - h_R R) g_{\mu\nu} - \alpha(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box) h_R = 8\pi G T_{\mu\nu}/c^4. \tag{3}$$

Here $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor and $h_R = \dfrac{dh}{dR}$.

Modified TOV equations in $f(R)$ gravity

Spherically symmetric solution

We are searching for the solutions of these equations assuming a spherically symmetric metric with two independent functions of radial coordinate, that is:

$$ds^2 = -e^{2\phi}c^2dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{4}$$

The energy–momentum tensor in the r.h.s. of Eq. (3) is that of a perfect fluid, i.e. $T = \mathrm{diag}(\rho c^2, -P, -P, -P)$, where ρ is the matter density and P is the pressure.
The components of the field equations are

$$\begin{aligned} -8\pi G\rho/c^2 &= -r^{-2} + e^{-2\lambda}(1-2r\lambda')r^{-2} + \alpha h_R(-r^{-2} + e^{-2\lambda}(1-2r\lambda')r^{-2}) \\ &\quad -\frac{1}{2}\alpha(h - h_R R) + e^{-2\lambda}\alpha[h'_R r^{-1}(2 - r\lambda') + h''_R], \end{aligned} \tag{5}$$

$$\begin{aligned} 8\pi GP/c^4 &= -r^{-2} + e^{-2\lambda}(1+2r\phi')r^{-2} + \alpha h_R(-r^{-2} + e^{-2\lambda}(1+2r\phi')r^{-2}) \\ &\quad -\frac{1}{2}\alpha(h - h_R R) + e^{-2\lambda}\alpha h'_R r^{-1}(2 + r\phi'), \end{aligned} \tag{6}$$

where prime denotes derivative with respect to radial distance, r.

Modified TOV equations in $f(R)$ gravity

For the exterior solution, a Schwarzschild solution is assumed. For this reason, it is convenient to define the change of variable (*Cooney et al.(2010)*)

$$e^{-2\lambda} = 1 - \frac{2GM}{c^2 r}\,. \tag{7}$$

The value of parameter M on the surface of a neutron stars can be considered as a gravitational star mass.
The useful relation

$$\frac{GdM}{c^2 dr} = \frac{1}{2}\left(1 - e^{-2\lambda}(1 - 2r\lambda')\right). \tag{8}$$

The hydrostatic condition equilibrium, $\nabla^\mu T_{\mu\nu} = 0$, for a perfect fluid is

$$\frac{dP}{dr} = -(\rho + P/c^2)\frac{d\phi}{dr}. \tag{9}$$

The second TOV equation can be obtained by substitution of the derivative $d\phi/dr$ from (9) in Eq.(6).

Modified TOV equations in $f(R)$ gravity

The dimensionless variables defined according to the substitutions are used.

$$M = mM_\odot, \quad r \to r_g r, \quad \rho \to \rho M_\odot / r_g^3, \quad P \to pM_\odot c^2 / r_g^3, \quad R \to R/r_g^2,$$

$$r_g = GM_\odot / c^2 = 1.47473 \text{ km}.$$

Eqs. (5), (6) can be rewritten as

$$\left(1 + \alpha r_g^2 h_R + \frac{1}{2}\alpha r_g^2 h'_R r\right)\frac{dm}{dr} = 4\pi\rho r^2 - \frac{1}{4}\alpha r^2 r_g^2 \left(h - h_R R - 2\left(1 - \frac{2m}{r}\right)\left(\frac{2h'_R}{r} + h''_R\right)\right), \tag{10}$$

$$8\pi p = -2\left(1 + \alpha r_g^2 h_R\right)\frac{m}{r^3} - \left(1 - \frac{2m}{r}\right)\left(\frac{2}{r}(1 + \alpha r_g^2 h_R) + \alpha r_g^2 h'_R\right)(\rho + p)^{-1}\frac{dp}{dr} - \tag{11}$$

$$-\frac{1}{2}\alpha r_g^2\left(h - h_R R - 4\left(1 - \frac{2m}{r}\right)\frac{h'_R}{r}\right), \quad {}' = d/dr.$$

Modified TOV equations in $f(R)$ gravity

Scalar degree of freedom

For non-zero α, one needs the third equation for the Ricci curvature scalar. The trace of field Eqs. (3) gives the relation

$$3\alpha\Box h_R + \alpha h_R R - 2\alpha h - R = -\frac{8\pi G}{c^4}(-3P + \rho c^2). \tag{12}$$

In dimensionless variables

$$\begin{gathered} 3\alpha r_g^2 \left(\left(\frac{2}{r} - \frac{3m}{r^2} - \frac{dm}{rdr} - \left(1 - \frac{2m}{r}\right) \frac{dp}{(\rho + p)dr} \right) \frac{d}{dr} + \left(1 - \frac{2m}{r}\right) \frac{d^2}{dr^2} \right) h_R + \alpha r_g^2 h_R R - \\ -2\alpha r_g^2 h - R = -8\pi(\rho - 3p). \end{gathered} \tag{13}$$

The EoS for matter inside star to the Eqs. (10), (11), (13) is needed. For the sake of simplicity, one can use the polytropic EoS $p \sim \rho^\gamma$ although a more realistic EoS has to take into account different physical states for different regions of the star and it is more complicated.

Neutron star models in $f(R)$ gravity

Perturbative approach

The solution of Eqs. (10)-(13) can be achieved by using a perturbative approach (see *Arapoglu et al.(2011)* for details):

$$p = p^{(0)} + \alpha p^{(1)} + \ldots, \quad \rho = \rho^{(0)} + \alpha \rho^{(1)} + \ldots, \quad m = m^{(0)} + \alpha m^{(1)} + \ldots, \quad R = R^{(0)} + \alpha R^{(1)} + \ldots \tag{14}$$

The functions $\rho^{(0)}$, $p^{(0)}$, $m^{(0)}$ and $R^{(0)}$ satisfy to standard TOV equations assumed at zeroth order. Terms containing h_R are assumed to be of first order in the small parameter α. We have, for the $m = m^{(0)} + \alpha m^{(1)}$ and for pressure $p = p^{(0)} + \alpha p^{(1)}$ the following equations

$$\frac{dm}{dr} = 4\pi\rho r^2 - \alpha r^2 \left(4\pi\rho^{(0)} h_R + \frac{1}{4}(h - h_R R)\right) + \frac{1}{2}\alpha \left(\left(2r - 3m^{(0)} - 4\pi\rho^{(0)} r^3\right) \frac{d}{dr} + r(r - 2m^{(0)}) \frac{d^2}{dr^2} \right. \tag{15}$$

$$\frac{r - 2m}{\rho + p} \frac{dp}{dr} = 4\pi r^2 p + \frac{m}{r} - \alpha r^2 \left(4\pi p^{(0)} h_R + \frac{1}{4}(h - h_R R)\right) - \alpha \left(r - 3m^{(0)} + 2\pi p^{(0)} r^3\right) \frac{dh_R}{dr}. \tag{16}$$

The Ricci curvature scalar, in terms containing h_R and h, has to be evaluated at $\mathcal{O}(1)$ order, i.e.

$$R \approx R^{(0)} = 8\pi(\rho^{(0)} - 3p^{(0)})\,. \tag{17}$$

These equations have the same analytical representation:

$$\zeta = \frac{a_1 + a_2\xi + a_3\xi^3}{1 + a_4\xi} f(a_5(\xi - a_6)) + (a_7 + a_8\xi) f(a_9(a_{10} - \xi)) + (a_{11} + a_{12}\xi) f(a_{13}(a_{14} - \xi)) + \quad (18)$$

$$+ (a_{15} + a_{16}\xi) f(a_{17}(a_{18} - \xi)), \quad \zeta = \log(P/\mathrm{dyncm}^{-2}), \qquad \xi = \log(\rho/\mathrm{gcm}^{-3}), \qquad f(x) = \frac{1}{\exp(x) + 1}.$$

The coefficients a_i for SLy and FPS EoS are given in *Camenzind(2007)*.
The model of neutron star with a quark core. The quark matter can be described by the very simple EoS:

$$p_Q = a(\rho - 4B), \quad a = \mathrm{const}, \quad B \sim 60 - 90\mathrm{Mev/fm}^3 \qquad (19)$$

The value $a = 0.28$ correspons to $m_s = 250$ Mev. For numerical calculations, Eq. (19) is used for $\rho \geq \rho_{tr}$, where ρ_{tr} is the transition density for which $p_Q = p_{FPS}$ or p_{SLy}. For FPS equation, the transition density is $\rho_{tr} = 1.069 \times 10^{15}$ g/cm^3 ($B = 80$ Mev/fm^3), for SLy equation $\rho_{tr} = 1.029 \times 10^{16}$ g/cm^3 ($B = 60$ Mev/fm^3).

The model of quadratic gravity with logarithmic corrections in curvature (*Nojiri and Odintsov(2004)*):

$$f(R) = R + \alpha R^2 (1 + \beta \ln(R/\mu^2)), \tag{20}$$

where $|\alpha| < 1$ (in units r_g^2) and the dimensionless parameter $|\beta| < 1$. This model is considered in (*Alavirad and Weller(2012)*) for SLy equation. However it is not valid beyond the point $R = 0$ and we cannot apply our analysis for stars with central density $\rho_c > 1.72 \times 10^{15}$ g/cm^3 (for SLy equation) and for $\rho_c > 2.35 \times 10^{15}$ g/cm^3 (for FPS equation). The maximal mass of neutron star at various values α and β is close to the corresponding one in General Relativity at these critical densities (for FPS - $1.75 M_\odot$, for SLy - $1.93 M_\odot$). On the other hand, for model with quark core, the condition $R = 8\pi(\rho^{(0)} - 3p^{(0)}) > 0$ is satisfied at arbitrary densities. The analysis shows that maximal mass is decreasing with growing α. By using a piecewise EoS (FPS+quark core) one can obtain stars with radii ~ 9.5 km and masses $\sim 1.50 M_\odot$. In contrast with General Relativity, the minimal radius of neutron star for this equation is 9.9 km.

Neutron star models in $f(R)$ gravity

Model I

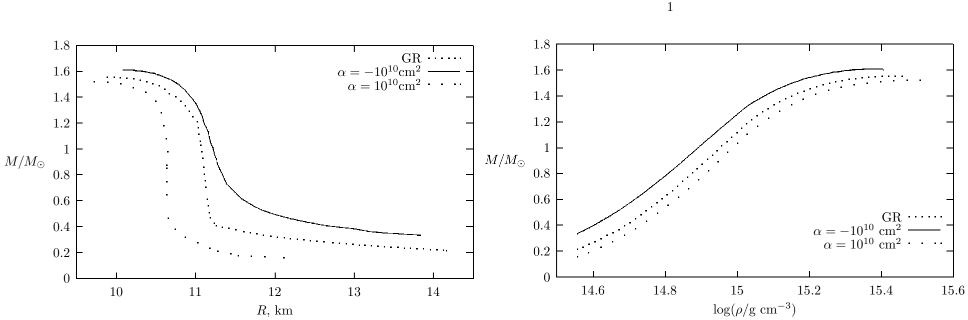

Figure : The mass-radius diagram (left panel) and the dependence of neutron star mass from central density (right panel) for neutron stars in $f(R)$ model with a logarithmic correction (20) compared with General Relativity by using a piecewise FPS+QC equation of state.

Neutron star models in $f(R)$ gravity

Model II

It is interesting to investigate also the R^2 model with a cubic correction:

$$f(R) = R + \alpha R^2(1 + \gamma R)\,. \tag{21}$$

The case where $|\gamma R| \sim \mathcal{O}(1)$ for large R is more interesting. In this case the cubic term comparable with quadratic term. Of course we consider the case when $\alpha R^2(1 + \gamma R) << R$. In this case the perturbative approach is valid although the cubic term can exceed the value of quadratic term. For small masses, the results coincides with R^2 model. For narrow region of high densities the mass of neutron star is close to the analogue mass in General Relativity with $dM/d\rho_c > 0$. This means that this configuration is stable. For $\alpha = 5 \times 10^9$ cm^2, $\gamma = -10$ (in units r_g^2) the maximal mass of neutron star at high densities $\rho > 3.7 \times 10^{15}$ g/cm^3 is nearly $1.88 M_\odot$ and radius is about ~ 9 km (SLy equation). For $\gamma = -20$ the maximal mass is $1.94 M_\odot$ and radius is about ~ 9.2 km. In the General Relativity, for SLy equation, the minimal radius of neutron stars is nearly 10 km. Therefore such a model of $f(R)$ gravity can give rise to neutron stars with smaller radii than in General Relativity. For FPS equation of state, we have the similar situation. Therefore such theory can describe (assuming only the SLy equation), the existence of peculiar neutron stars with mass $\sim 2M_\odot$ (the measured mass of PSR J1614-2230 *Ozel et al.(2010)*) and compact stars ($R \sim 9$ km) with masses $M \sim 1.6 - 1.7 M_\odot$ (see *Ozel et al.(2009)*, *Guver et al.(2010)*).

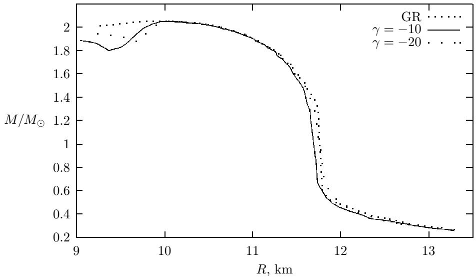

Figure : The mass-radius diagram for neutron stars in $f(R)$ model with cubic corrections (21)($\alpha = 5 \times 10^9$ cm^2) in comparison with General Relativity assuming a SLy equation of state. The parameters of stable configurations on the second "branch" of stability are $1.80 M_\odot < M < 1.89 M_\odot$, $9.04 < R < 9.36$ km and $2.95 \times 10^{15} < \rho_c < 3.46 \times 10^{15}$ g/cm^3 for $\gamma = -10$ and $1.87 M_\odot < M < 1.94 M_\odot$, $9.23 < R < 9.7$ km and $3.46 \times 10^{15} < \rho_c < 3.89 \times 10^{15}$ g/cm^3 for $\gamma = -20$. The parameter γ is measured in units r_g^2. For comparison we depicted the $M - R$ relation in GR also for the $dM/d\rho_c < 0$. One note that at given values of γ the cubic term is greater than quadratic only at high densities. For $\gamma = -20$ at $\rho > 2.5 \times 10^{15}$ g/cm^3 (that corresponds to central regions of star) and its maximum value $\gamma R^3|_{max} \sim 3R^2$. The perturbative approach is valid at these values of parameters.

Conclusion

We considered the mass-radius relations for neutron stars for the R^2 models with logarithmic and cubic corrections. We also investigated the dependence of the maximal mass from the central density of the structure. In the case of quadratic gravity with logarithmic corrections, assuming a piecewise equation of state (FPS+quark core), one obtains stellar objects with radii ~ 9.5 km and masses $\sim 1.50 M_\odot$. In contrast with General Relativity, the neutron star minimal radius for this equation is 9.9 km. In the case of quadratic gravity with cubic corrections, we found that, for high central densities ($\rho > 10\rho_{ns}$, where $\rho_{ns} = 2.7 \times 10^{14}$ g/cm^3 is the nuclear saturation density) stable star configurations exist. This effect gives rise to more compact stars than in General Relativity and could be extremely relevant from an observational point of view. In fact, it is interesting to note that using an equation of state in the framework of $f(R)$ gravity with cubic term gives rise to two important features: the existence of an upper limit on neutron star mass ($\sim 2M_\odot$) and the existence of neutron stars with radii $R \sim 9 - 9.5$ km and masses $\sim 1.7 M_\odot$. These facts could have a twofold interest: from one side, the approach could be useful to explain peculiar objects that evade explanation in the framework of standard General Relativity (e.g. the magnetars) and, from the other side, it could constitute a very relevant test for alternative gravities.

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