Symmetry Energy and Surface Properties of Neutron-Rich Exotic Nuclei

[1]M.K. Gaidarov, [2]P. Sarriguren, [1]A.N. Antonov,

[3]E. Moya de Guerra

[1]Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
[2]Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain
[3]Departamento de Fisica Atomica, Molecular y Nuclear, Facultad de Ciencias Fisicas, Universidad Complutense de Madrid, E-28040 Madrid, Spain

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Outline

Motivation:

- Structure of exotic nuclei, dynamics of heavy-ion reactions, physics of giant collective excitations, physics of neutron stars
- Study of different equilibrium configurations of atomic nuclei (g.s.) and transition regions between them
- ^{208}Pb: linear correlation between the neutron skin thickness and the slope of the neutron EOS at $\rho \approx 0.10$ fm^{-3}

Theoretical approaches:

- Deformed HF+BCS formalism with Skyrme forces (SLy4, SGII, Sk3, LNS)
- Coherent density fluctuation model (CDFM)
- Brueckner energy-density functional for assymetric nuclear matter (ANM)

Results: Ni (A=74-84), Kr (A=82-96; 96-120), Sn (A=124-152), Sm (A=140-156) and Pb (A=202-214) isotopes

- Symmetry energy (s), pressure (p_0), asymmetric compressibility (ΔK)
- Correlation between the neutron skin thickness (ΔR) and s, p_0, ΔK
- Density dependence of the nuclear symmetry energy: ΔR vs p_0 for a given nucleus
- Arguments in proof of the existence of kinks in Ni and Sn isotopic chains, but not in the Pb chain

Papers:

M.K. Gaidarov et al., Phys. Rev. *C* **84**, 034316 (2011)

Phys. Rev. *C* **85**, 064319 (2012)

The key EOS parameters in ANM

$$E(\rho,\delta) = E(\rho,0) + s(\rho)\delta^2 + O(\delta^4) + \dots$$

$$\delta = (\rho_n - \rho_p)/\rho$$

$$E(\rho,0) = E_0 + \frac{K}{18\rho_0^2}(\rho-\rho_0)^2 + \dots$$

$$s^{ANM}(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\bigg|_{\delta=0} =$$

$$= a_4 + \frac{p_0^{ANM}}{\rho_0^2}(\rho-\rho_0) + \frac{\Delta K^{ANM}}{18\rho_0^2}(\rho-\rho_0)^2 + \dots$$

a_4 is the symmetry energy at $\rho = \rho_0$

Weizsacker mass formula

$$E(N,Z) = E_{mac} + E_{mic} = E_V + E_S + E_a + E_C + E_{mic} =$$

$$= -a_V A + a_S A^{2/3} + a_a \frac{(N-Z)^2}{A} + a_C \frac{Z^2}{A^{1/3}} + E_{mic}$$

Symmetry pressure

$$p_0^{ANM} = \rho_0^2 \frac{\partial s^{ANM}}{\partial\rho}\bigg|_{\rho=\rho_0}$$

Slope parameter

$$L^{ANM} = \frac{3p_0^{ANM}}{\rho_0}$$

Asymmetric compressibility

$$\Delta K^{ANM} = 9\rho_0^2 \frac{\partial^2 s^{ANM}}{\partial\rho^2}\bigg|_{\rho=\rho_0}$$

The coherent density fluctuation model (CDFM)

$$\rho_x(\vec{r}) = \rho_0(x)\Theta(x - |\vec{r}|), \text{ where } \rho_0(x) = \frac{3A}{4\pi x^3}$$

$$\rho_x(\vec{r},\vec{r}\,') = 3\rho_0(x)\frac{j_1(k_F(x)|\vec{r}-\vec{r}\,'|)}{(k_F(x)|\vec{r}-\vec{r}\,'|)}\Theta\left(x - \frac{|\vec{r}+\vec{r}\,'|}{2}\right), \text{ where}$$

j_1 is the first-order spherical Bessel function

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\alpha}{x} \text{ with } \alpha = \left(\frac{9\pi A}{8}\right)^{1/3} \cong 1.52A^{1/3} \text{ is the Fermi momentum}$$

$$\rho(\vec{r},\vec{r}\,') = \int_0^\infty dx|F(x)|^2 \rho_x(\vec{r},\vec{r}\,')$$

$$W(\vec{r},\vec{k}) = \int_0^\infty dx|F(x)|^2 W_x(\vec{r},\vec{k}), \text{ where } W_x(\vec{r},\vec{k}) = \frac{4}{(2\pi)^3}\Theta(x - |\vec{r}|)\,\Theta(k_F(x) - |\vec{k}|)$$

$$\rho(r) = \int d\vec{k} W(\vec{r},\vec{k}) = \int_0^\infty dx \left|F(x)\right|^2 \frac{3A}{4\pi x^3} \Theta(x - |\vec{r}|); \quad \int \rho(\vec{r}) d\vec{r} = A$$

In the case of monotonically decreasing local densities ($d\rho(r)/dr \le 0$)

$$\left|F(x)\right|^2 = -\frac{1}{\rho_0(x)} \frac{d\rho(r)}{dr}\bigg|_{r=x}; \quad \int_0^\infty dx \left|F(x)\right|^2 = 1$$

Brueckner energy-density functional for infinite NM

$V(x) = AV_0(x) + V_C - V_{CO}$, where

$$V_0(x) = 37.53\left[(1+\delta)^{5/3} + (1-\delta)^{5/3}\right]\rho_0^{2/3}(x) +$$

$$+ b_1\rho_0(x) + b_2\rho_0^{4/3}(x) + b_3\rho_0^{5/3}(x) + \delta^2\left[b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x)\right]$$

$b_1 = -741.28$; $b_2 = 1179.89$; $b_3 = -467.54$; $b_4 = 148.26$; $b_5 = 372.84$; $b_6 = -769.57$

$$V_C = \frac{3}{5}\frac{Z^2 e^2}{x} \qquad V_{CO} = 0.7386\, Ze^2 (3Z/4\pi x^3)^{1/3}$$

$$S^{ANM}(x) = 41.7\rho_0^{2/3}(x) + b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x)$$

$$p_0^{ANM}(x) = 27.8\rho_0^{5/3}(x) + b_4\rho_0^{2}(x) + \frac{4}{3}b_5\rho_0^{7/3}(x) + \frac{5}{3}b_6\rho_0^{8/3}(x)$$

$$\Delta K^{ANM}(x) = -83.4\rho_0^{2/3}(x) + 4b_5\rho_0^{4/3}(x) + 10b_6\rho_0^{5/3}(x)$$

Symmetry energy parameters of finite nuclei in CDFM

$$s = \int_0^\infty dx\,|F(x)|^2 S^{ANM}(x)$$

$$p_0 = \int_0^\infty dx\,|F(x)|^2 p_0^{ANM}(x)$$

$$\Delta K = \int_0^\infty dx\,|F(x)|^2 \Delta K^{ANM}(x)$$

Axially Deformed Skyrme Hartree-Fock Method

E. Moya de Guerra, P. Sarriguren, J. A. Caballero, M. Casas, and D. W. L. Sprung, Nucl. Phys. **A529**, 68 (1991)

$$\Phi_i\left(\vec{r}, \sigma, q\right) \; = \; \chi_{q_i}(q)\left[\Phi_i^+(r_\perp, z)e^{i\Lambda^- \varphi}\chi_+(\sigma) + \Phi_i^-(r_\perp, z)e^{i\Lambda^+ \varphi}\chi_-(\sigma)\right] \quad (1)$$

$$\Phi_i\left(\vec{r}, \sigma, q\right) = \chi_{q_i}(q)\sum_\alpha C_\alpha^i \phi_\alpha\left(\vec{r}, \sigma\right) \quad (2)$$

with $\alpha = \{n_\perp, n_z, \Lambda, \Sigma\}$

$$\phi_\alpha\left(\vec{r}, \sigma\right) = \psi_{n_\perp}^\Lambda(r_\perp)\psi_{n_z}(z)\frac{e^{i\Lambda\varphi}}{\sqrt{2\pi}}\chi_\Sigma(\sigma) \quad (3)$$

The spin-independent proton and neutron densities:

$$\rho(\vec{r}) = \rho(r_\perp, z) = \sum_i 2v_i^2 \rho_i(r_\perp, z) \tag{4}$$

$$\rho_i(\vec{r}) = \rho_i(r_\perp, z) = \left|\Phi_i^+(r_\perp, z)\right|^2 + \left|\Phi_i^-(r_\perp, z)\right|^2 \tag{5}$$

with

$$\Phi_i^\pm(r_\perp, z) = \frac{1}{\sqrt{2\pi}} \times \sum_\alpha \delta_{\Sigma,\pm 1/2}\, \delta_{\Lambda,\Lambda^\mp}\, C_\alpha^i\, \psi_{n_\perp}^\Lambda(r_\perp)\, \psi_{n_z}(z) \tag{6}$$

$$\int \rho(\vec{r}) d\vec{r} = X \tag{7}$$

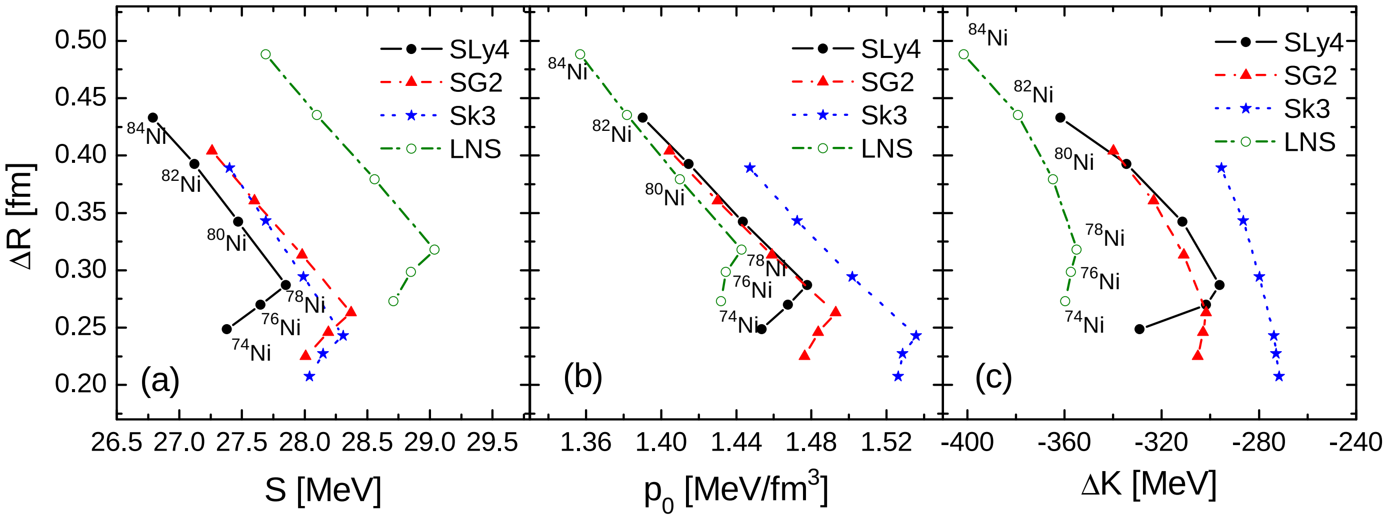
The mean square and rms radii:

$$< r_{\mathrm{p,n}}^2 >= \frac{\int r^2 \rho_{\mathrm{p,n}}(\vec{r}) d\vec{r}}{\int \rho_{\mathrm{p,n}}(\vec{r}) d\vec{r}} \tag{8}$$

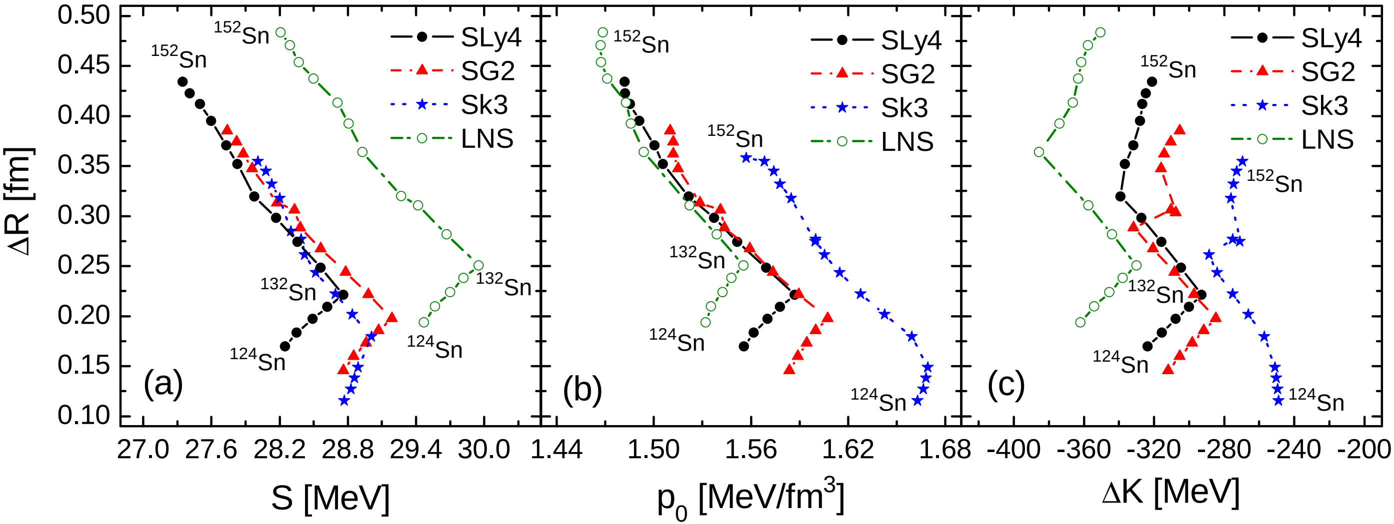
$$r_{\mathrm{p,n}} =< r_{\mathrm{p,n}}^2 >^{1/2} \tag{9}$$

The neutron skin thickness:

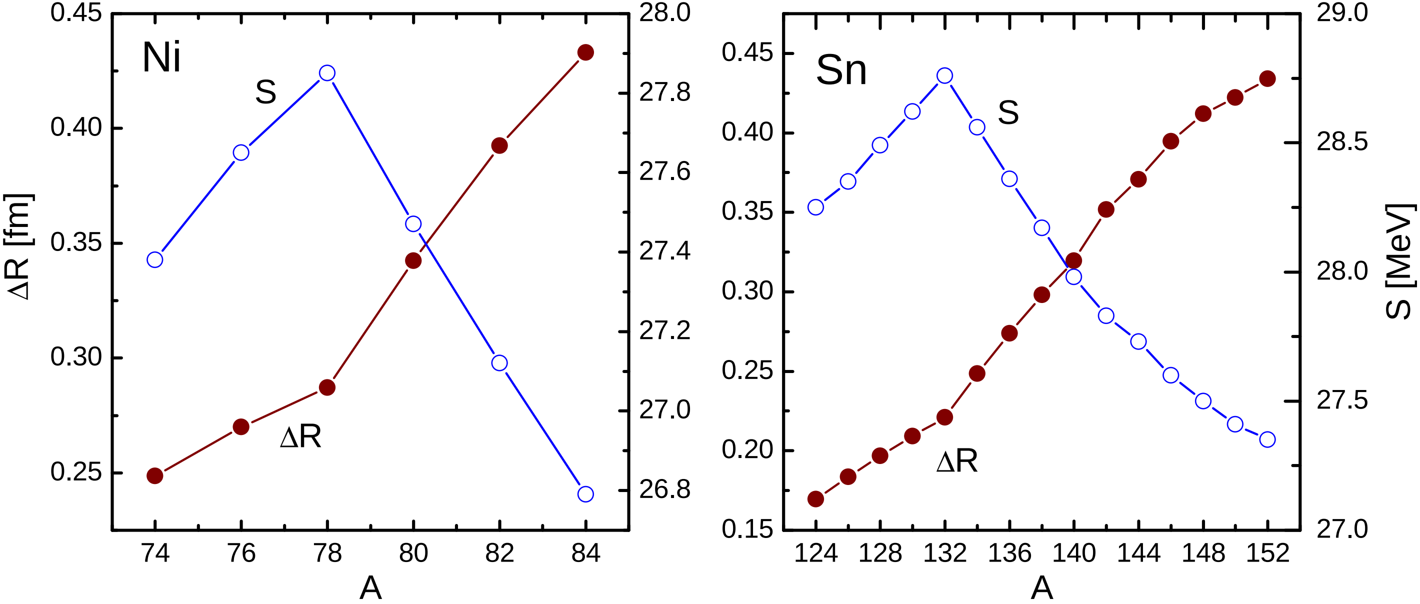
$$\Delta R =< r_{\mathrm{n}}^2 >^{1/2} - < r_{\mathrm{p}}^2 >^{1/2} \tag{10}$$



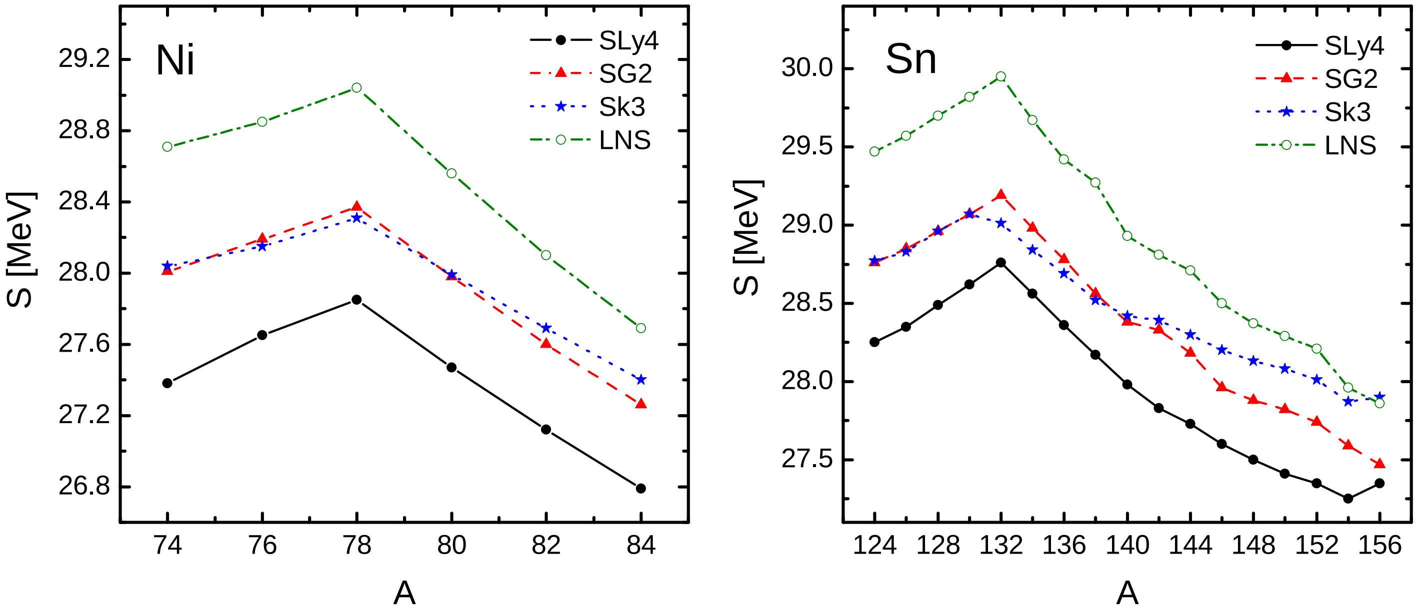
ΔR [fm]
S [MeV]
p_0 [MeV/fm^3]
ΔK [MeV]
SLy4
SG2
Sk3
LNS
(a)
(b)
(c)
^{84}Ni
^{82}Ni
^{80}Ni
^{78}Ni
^{76}Ni
^{74}Ni

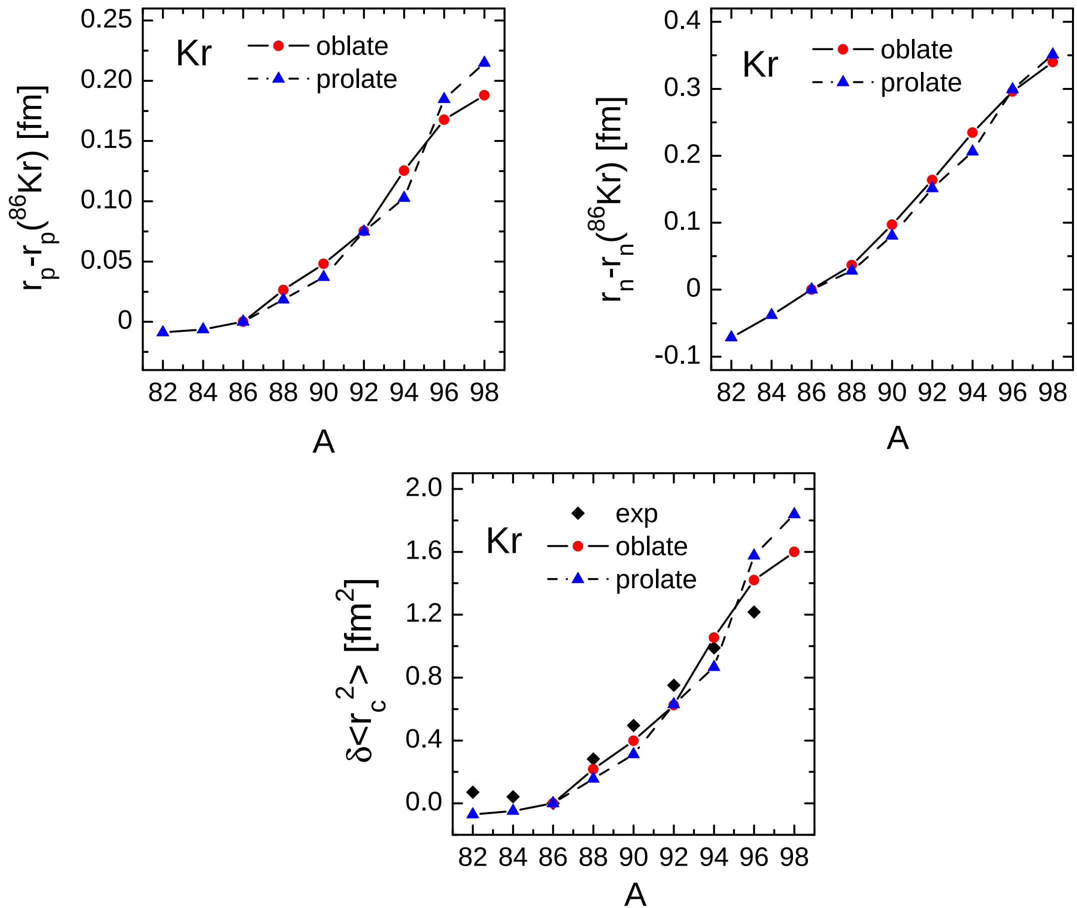


ΔR [fm]
S [MeV]
p_0 [MeV/fm^3]
ΔK [MeV]
(a)
(b)
(c)
SLy4
SG2
Sk3
LNS
^{152}Sn
^{132}Sn
^{124}Sn

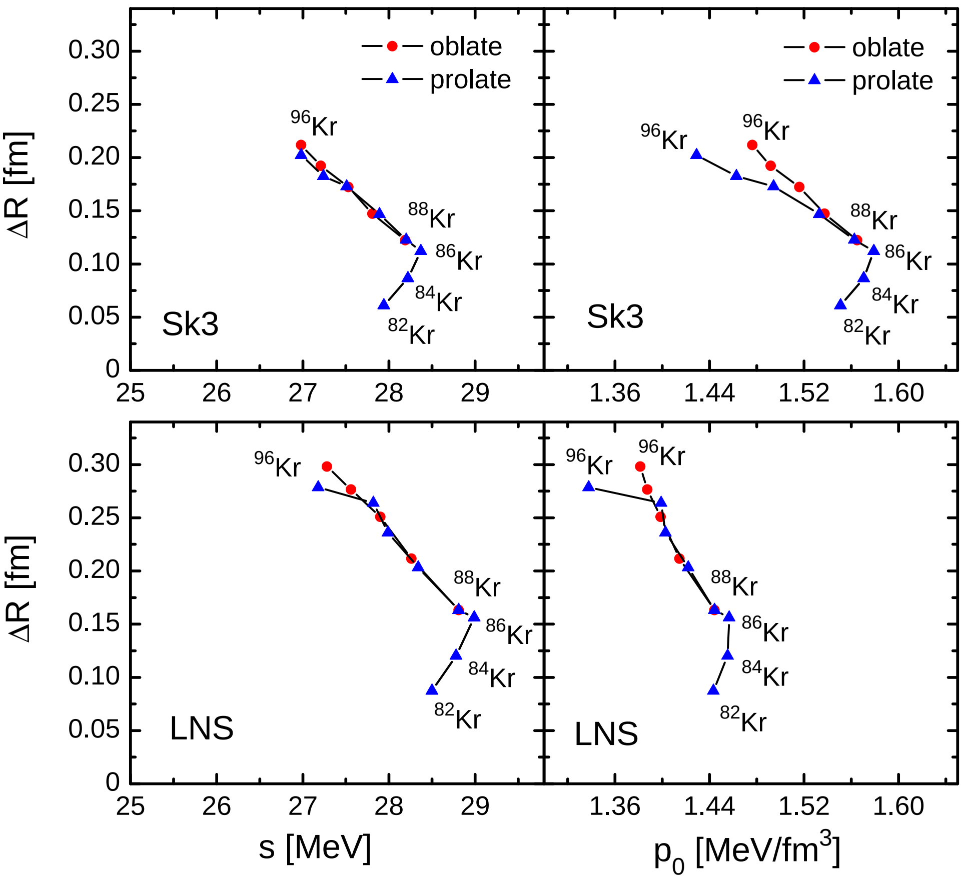


Ni
S
ΔR
ΔR [fm]
A
Sn
S
ΔR
S [MeV]
A

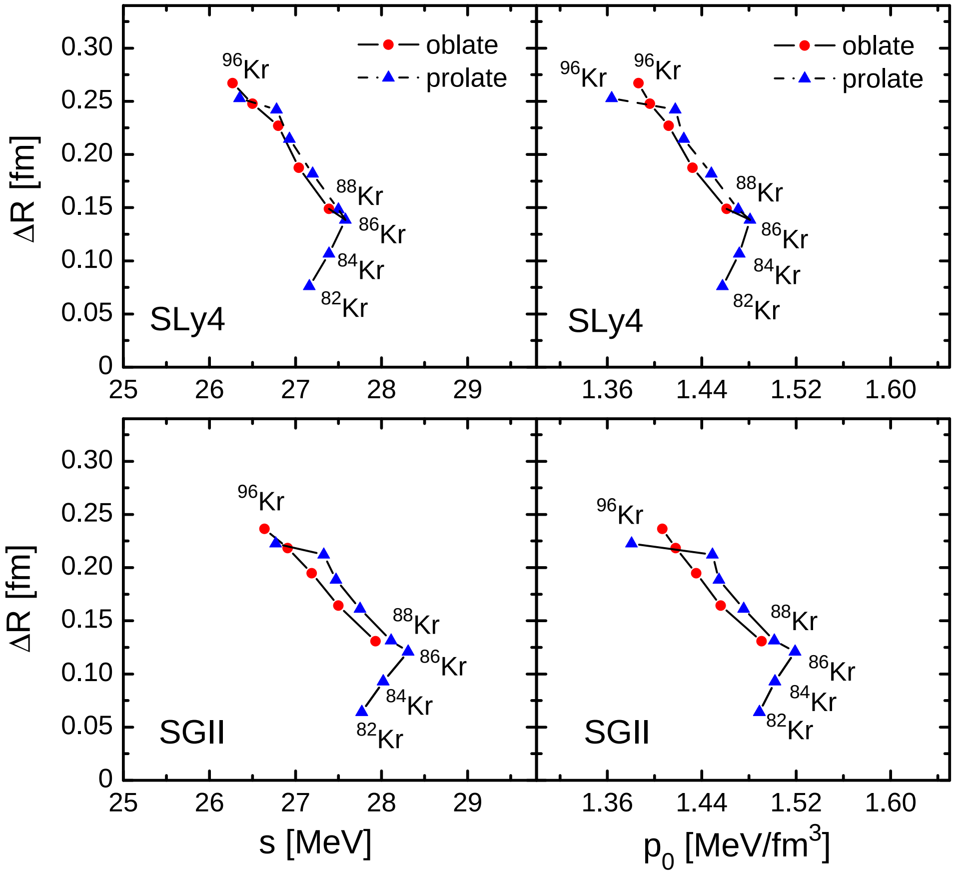

Ni
SLy4
SG2
Sk3
LNS
S [MeV]
A
Sn
SLy4
SG2
Sk3
LNS
S [MeV]
A



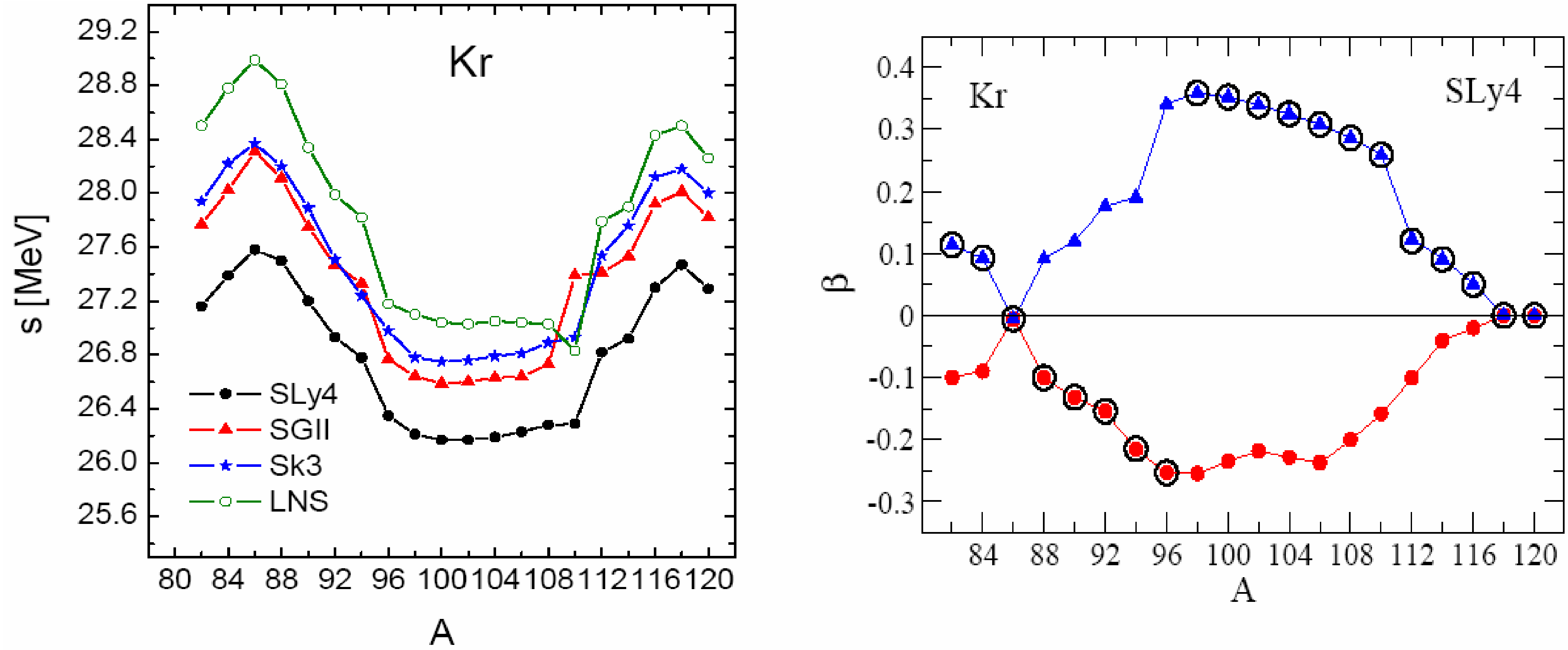
Kr
oblate
prolate
0.25
0.20
0.15
0.10
0.05
0
r_p-r_p(^{86}Kr) [fm]
82 84 86 88 90 92 94 96 98
A
Kr
oblate
prolate
0.4
0.3
0.2
0.1
0
-0.1
r_n-r_n(^{86}Kr) [fm]
82 84 86 88 90 92 94 96 98
A
Kr
exp
oblate
prolate
2.0
1.6
1.2
0.8
0.4
0.0
$\delta\langle r_c^2\rangle$ [fm^2]
82 84 86 88 90 92 94 96 98
A



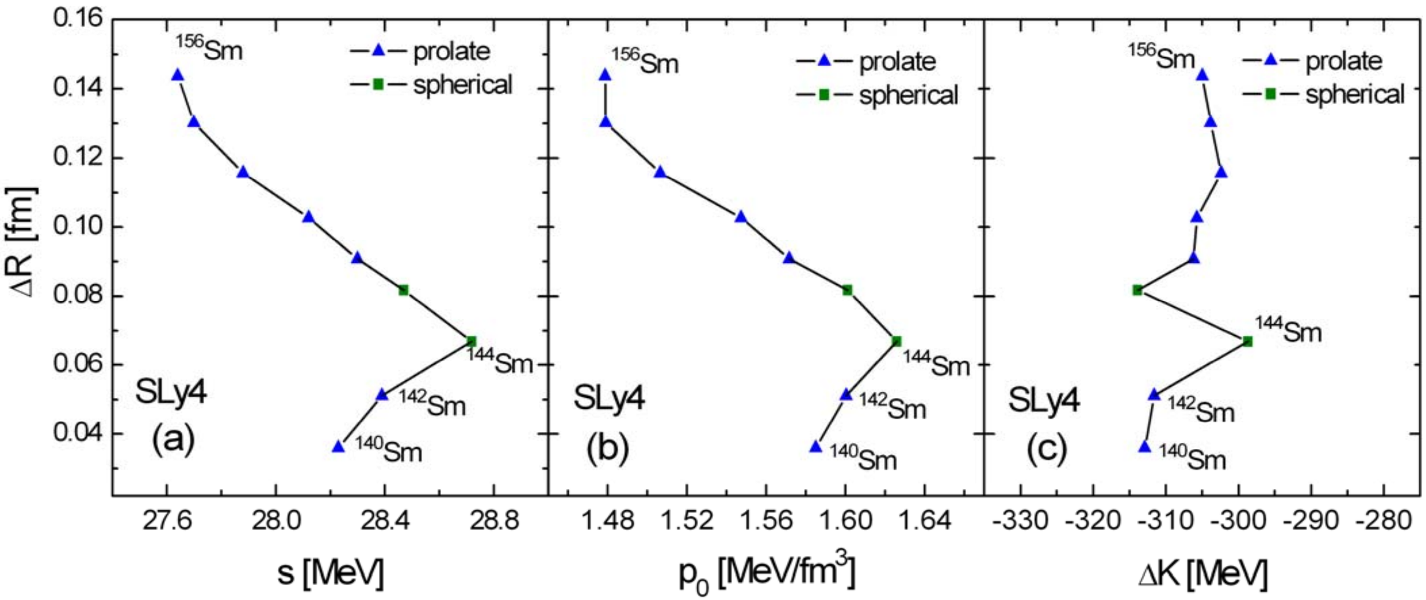
0.30
0.25
0.20
0.15
0.10
0.05
0
ΔR [fm]
oblate
prolate
96Kr
88Kr
86Kr
84Kr
82Kr
Sk3
LNS
25
26
27
28
29
1.36
1.44
1.52
1.60
s [MeV]
p0 [MeV/fm3]

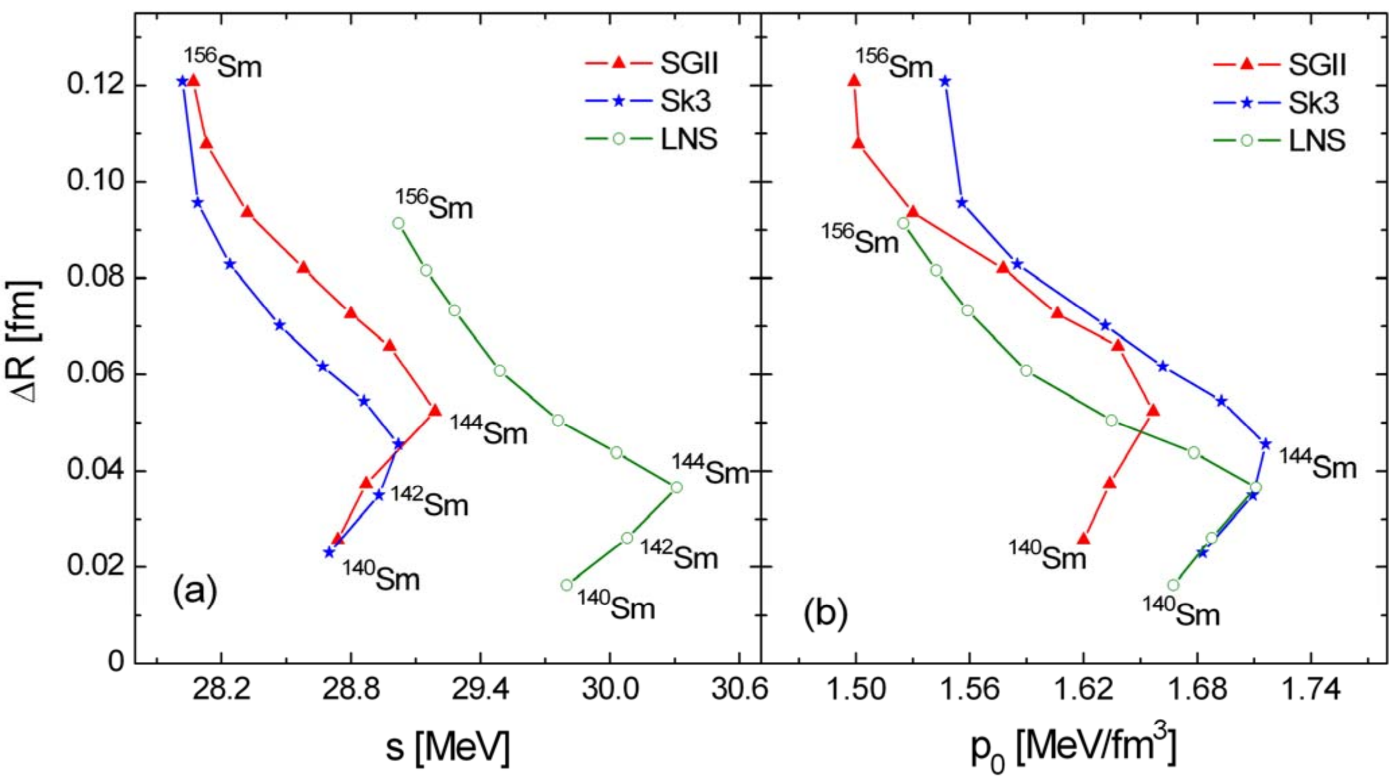


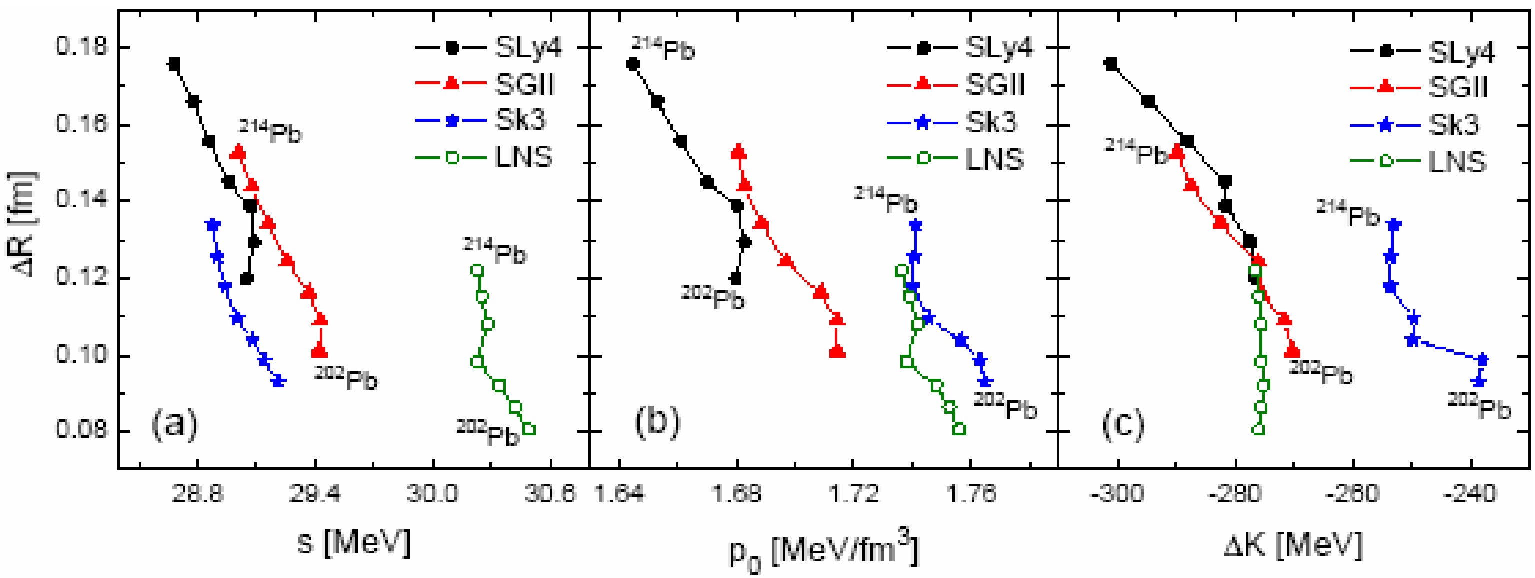
ΔR [fm]
0.30
0.25
0.20
0.15
0.10
0.05
0
oblate
prolate
^{96}Kr
^{88}Kr
^{86}Kr
^{84}Kr
^{82}Kr
SLy4
SGII
25
26
27
28
29
1.36
1.44
1.52
1.60
s [MeV]
p_0 [MeV/fm^3]



Kr
s [MeV]
A
SLy4
SGII
Sk3
LNS
Kr
SLy4
β
A


ΔR [fm]
s [MeV]
p0 [MeV/fm3]
ΔK [MeV]
prolate
spherical
SLy4
(a)
(b)
(c)
156Sm
144Sm
142Sm
140Sm


ΔR [fm]
s [MeV]
p0 [MeV/fm3]
SGII
Sk3
LNS
(a)
(b)
156Sm
144Sm
142Sm
140Sm

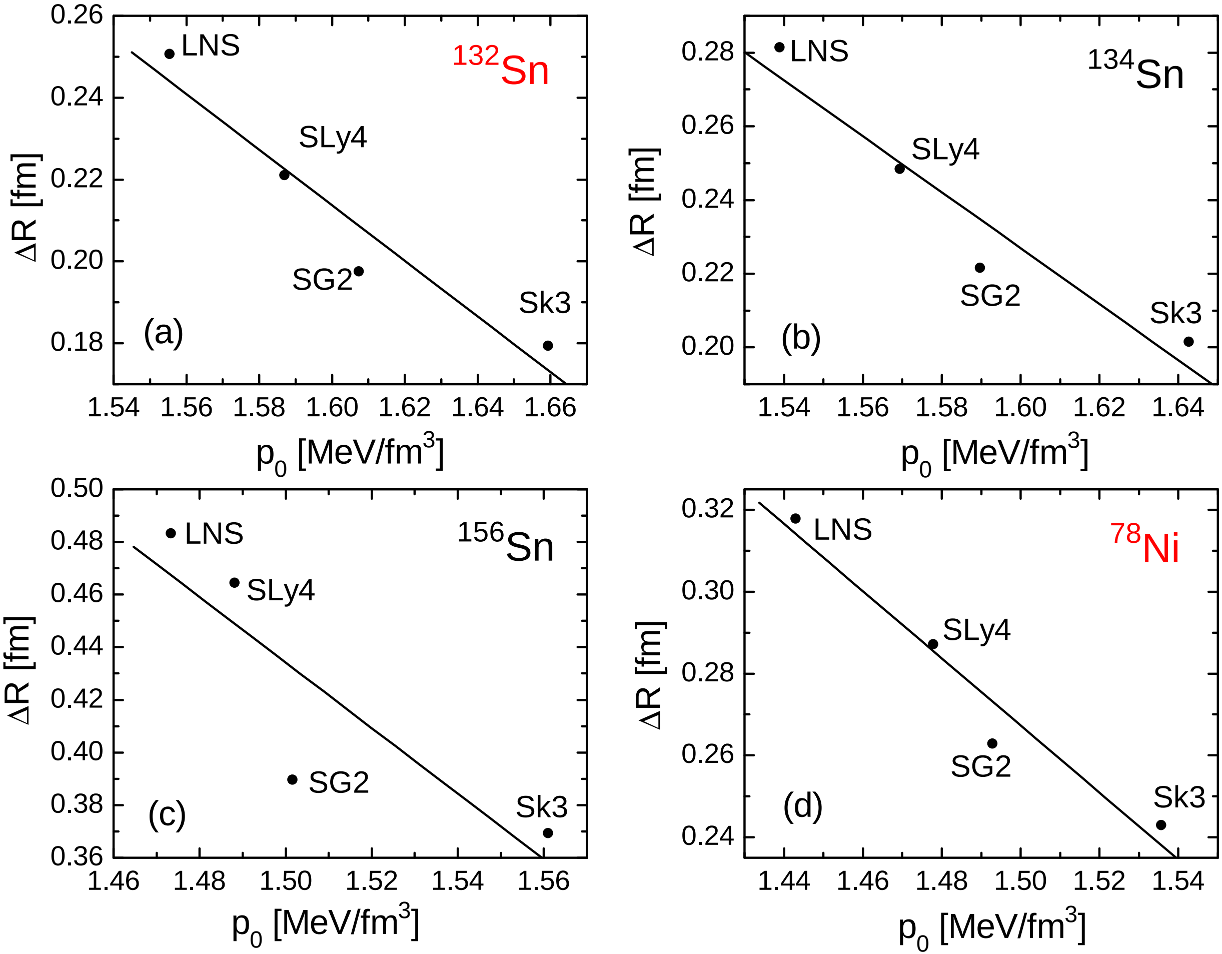


Neutron skin thickness of ^{208}Pb

Our result with SLy4 force: ΔR=0.1452 fm

JLAB (PREX Collaboration): $\Delta R = 0.33^{+0.16}_{-0.18}$ fm

Correlation with the dipole polarizability: $\Delta R = 0.156^{+0.025}_{-0.021}$ fm
(SV-min Skyrme functional)



ΔR [fm]
p$_0$ [MeV/fm^3]
^{132}Sn
(a)
LNS
SLy4
SG2
Sk3
0.26
0.24
0.22
0.20
0.18
1.54
1.56
1.58
1.60
1.62
1.64
1.66
^{134}Sn
(b)
0.28
0.26
0.24
0.22
0.20
1.54
1.56
1.58
1.60
1.62
1.64
^{156}Sn
(c)
0.50
0.48
0.46
0.44
0.42
0.40
0.38
0.36
1.46
1.48
1.50
1.52
1.54
1.56
^{78}Ni
(d)
0.32
0.30
0.28
0.26
0.24
1.44
1.46
1.48
1.50
1.52
1.54

- The quantity

$$\Delta s_{\pm} = \frac{s_{A\pm 2} - s_A}{s_A} \tag{1}$$

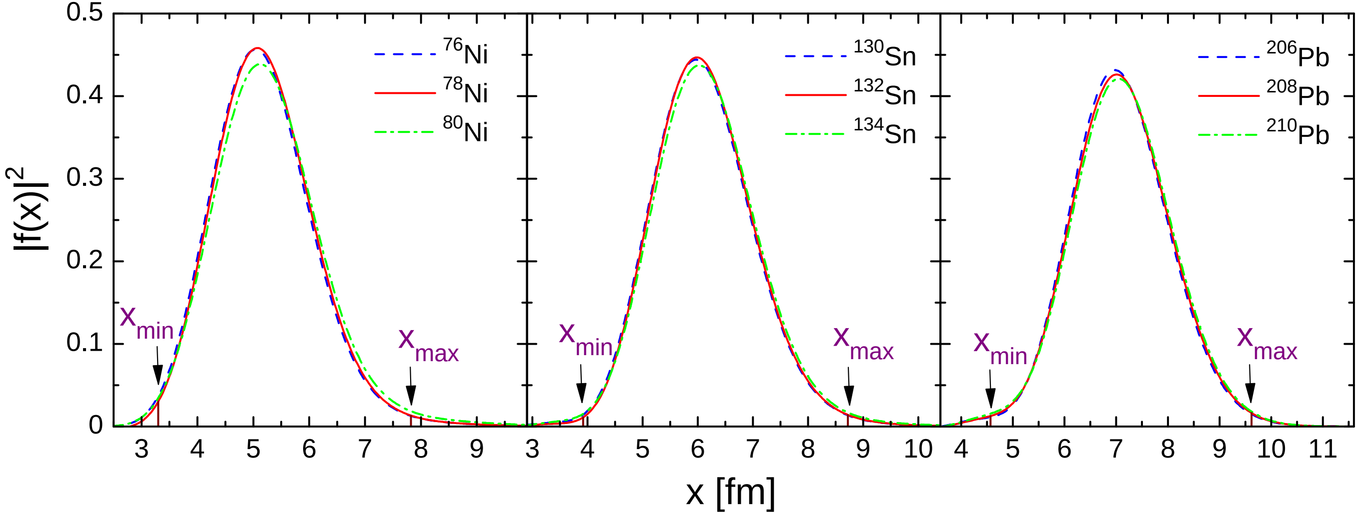
gives information on the relative deviation of the symmetry energy s of even-even isotopes with respect to the double-magic ones, namely with $A = 78$ for Ni, $A = 132$ for Sn, and $A = 208$ for Pb.

$$s = \int_0^{\infty} dx|f(x)|^2 s^{ANM}(x) \tag{2}$$

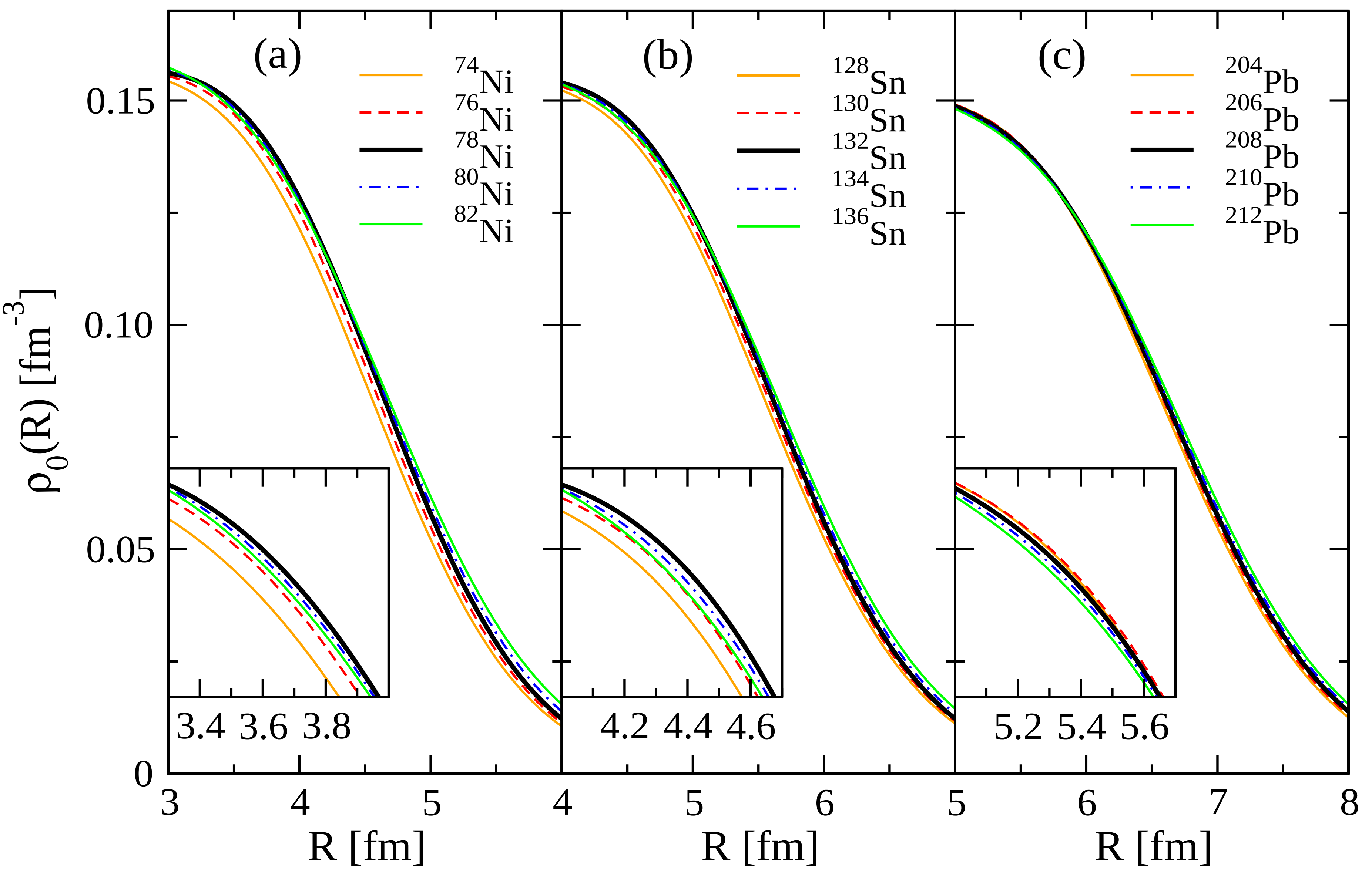
$x_{\text{min}} \rightarrow s^{ANM}(x)$ changes sign from negative (at $x < x_{\text{min}}$) to positive (at $x > x_{\text{min}}$)
$x_{\text{max}} \rightarrow$ if Δx=$x_{\text{max}} - x_{\text{min}}$, then $s - s_{\Delta x} \leq 0.1$ MeV

Relative deviation values of the symmetry energy Δs_{+} and Δs_{-} for the range of integration Δx and for Ni, Sn, and Pb isotopes.

	Ni	Sn	Pb
Δs_{+}	-0.0137	-0.0070	-0.0035
Δs_{-}	-0.0072	-0.0049	0.0038



0.5
0.4
0.3
0.2
0.1
0
|f(x)|2
76Ni
78Ni
80Ni
130Sn
132Sn
134Sn
206Pb
208Pb
210Pb
xmin
xmax
3 4 5 6 7 8 9
3 4 5 6 7 8 9 10
4 5 6 7 8 9 10 11
x [fm]



(a)
(b)
(c)
74Ni
76Ni
78Ni
80Ni
82Ni
128Sn
130Sn
132Sn
134Sn
136Sn
204Pb
206Pb
208Pb
210Pb
212Pb
ρ0(R) [fm-3]
0.15
0.10
0.05
0
3.4 3.6 3.8
4.2 4.4 4.6
5.2 5.4 5.6
3 4 5 4 5 6 5 6 7 8
R [fm]
R [fm]
R [fm]

Conclusions

- ✓ Deformed HF+BCS method with Skyrme-type density-dependent effective interaction + CDFM
- ✓ Three chains of spherical Ni, Sn, Pb and two chains of Kr and Sm deformed neutron-rich isotopes
- ✓ Four Skyrme parametrizations: SGII, Sk3, SLy4, and LNS
- ✓ Calculated: nuclear symmetry energy ***s***, neutron pressure p_0, asymmetric compressibility ***ΔK***, neutron skin thickness ***ΔR***

- ❑ CDFM: an alternative way for a transition from the properties of NM to the properties of finite nuclei.
- ❑ Brueckner EDF for infinite nuclear matter

- ❑ There exists an approximate linear correlation between ***ΔR*** and ***S***, ***ΔR*** and p_0 , while the relation between ***ΔR*** and ***ΔK*** is less pronounced. A behavior containing an inflexion point transition at specific shell closure is observed for these correlations (**^{78}Ni**, **^{132}Sn,** **^{86}Kr**, and **^{144}Sm**).

- Our HF+BCS calculations lead to ***s*** in the range of 27-30 MeV, which is in agreement with the empirical value of 30 ± 4 MeV. The calculated values of p_0=1.36-1.68 MeV/fm^3 lead to values of the slope parameter ***L***=26-32 MeV, in agreement with other theoretical predictions.

- The kinks displayed by the Ni and Sn can be understood as consequences of particular differences in the structure of these nuclei and the resulting densities and weight functions. It is shown that for the Pb isotopes the different signs of the relative deviations corresponding to the range of integration on ***x*** that contains the peak of $|f(x)|^2$ is in favor of the absence of kink in the Pb chain.

The capability of the present method can be further demonstrated by taking into consideration Skyrme-type and relativistic nuclear energy-density functionals.

$$E = \int d\mathbf{r}\rho(r)\bar{\varepsilon}[\rho(r)] \tag{1}$$

$$\begin{aligned} E &= \int d\mathbf{r} \int_0^\infty dx |f(x)|^2 \Theta(x-r)\rho_0(x)\bar{\varepsilon}[\rho(r)] \\ &= \int_0^\infty dx |f(x)|^2 \rho_0(x) \int_0^x 4\pi r^2 dr \bar{\varepsilon}[\rho(r)] \end{aligned} \tag{2}$$

Approximation 1:

$$|f(x)|^2 = \delta(x-r) \tag{3}$$

$$\rho(r) = \rho_0(x) = \frac{3A}{4\pi x^3} \tag{4}$$

$$E \simeq \int_0^\infty dx |f(x)|^2 A\bar{\varepsilon}[\rho_0(x)] \tag{5}$$

$$\begin{aligned} E &= \int_0^\infty dx|f(x)|^2 A\bar{\varepsilon}[\rho_0(x),\delta] \\ &= \int_0^\infty dx|f(x)|^2 A\{\bar{\varepsilon}[\rho_0(x),0] + S^{ANM}[\rho_0(x)]\delta^2 + O(\delta^4) + ...\}, \end{aligned} \quad (6)$$

where

$$\begin{aligned} S^{ANM}[\rho_0(x)] &= \frac{1}{2}\frac{\partial^2\bar{\varepsilon}[\rho_0(x),\delta]}{\partial\delta^2}\bigg|_{\delta=0} \\ &= 41.7\rho_0^{2/3}(r) + b_4\rho_0(r) + b_5\rho_0^{4/3}(r) + b_6\rho_0^{5/3}(r) \end{aligned} \quad (7)$$

For finite nucleus:

$$s = \int_0^\infty dx|f(x)|^2 AS^{ANM}[\rho_0(x)] \quad (8)$$

Approximation 2:

$$|f(x)|^2 = \delta(x-r), \quad \rho(r) = \rho_0(x) \tag{9}$$

$$s = S^{ANM}[\rho_0(r)] = S^{ANM}[\rho_0(x)] = a_4 \tag{10}$$

$$E = A\bar{\varepsilon}[\rho_0(x)] \tag{11}$$

No approximations:

$$s = \int_0^\infty dx|f(x)|^2\rho_0(x)\int_0^x 4\pi r^2 dr S[\rho(r)], \tag{12}$$

where

$$S[\rho(r)] = 41.7\rho^{2/3}(r) + b_4\rho(r) + b_5\rho^{4/3}(r) + b_6\rho^{5/3}(r) \tag{13}$$

$$\begin{aligned} \rho(r) &= \int_0^\infty dx'|f(x')|^2\Theta(x'-r)\frac{3A}{4\pi x'^3} \\ &= \int_r^\infty dx'|f(x')|^2\frac{3A}{4\pi x'^3} \end{aligned} \tag{14}$$