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Q^2-evolution of parton densities at small x values. Combined H1 and ZEUS F_2 data.

OUTLINE

1. Introduction
2. Results: flat initial conditions for PDFs at small x values
   (i.e. min information about initial conditions, or min contribution from initial conditions.)
3. Conclusions and Prospects

1. Introduction to DIS

A. Deep-inelastic scattering cross-section:

$$\sigma \sim L^{\mu\nu} F^{\mu\nu}$$

Hadron part $F^{\mu\nu}$ ($Q^2 = -q^2 > 0$, $x = Q^2/[2(pq)]$):

$$\begin{aligned} F^{\mu\nu} &= (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) F_1(x, Q^2) \\ &- (p^\mu - \frac{(pq)}{q^2} q^\mu)(p^\nu - \frac{(pq)}{q^2} q^\nu) \frac{2x}{q^2} F_2(x, Q^2) + ..., \end{aligned}$$

where $F_k(x, Q^2)$ ($k = 1, 2, 3, L$) - are DIS SF and q and p are photon and hadron (parton) momentums.

B. Wilson operator expansion: Mellin moments $M_k(n, Q^2)$ of DIS SF $F_k(x, Q^2)$ can be represented as sum

$$M_k(n, Q^2) = \sum_{a=NS,SI,g} \underbrace{C_k^a(n, Q^2/\mu^2)}_{\text{Coeff. function}} A_a(n, \mu^2),$$

where $A_a(n, \mu^2) =< N|\mathcal{O}^a_{\mu_1,\ldots,\mu_n}|N >$ are matrix elements of the Wilson operators $\mathcal{O}^a_{\mu_1,\ldots,\mu_n}$.

C. The matrix elements $A_a(n,\mu^2)$ are Mellin moments of the unpolarized PD $f_a(n,\mu^2)$.

DGLAP [= Renormgroup] equations:

$$\frac{d}{d\ln Q^2} f_a(x,Q^2) = \int_x^1 \frac{dy}{y} \sum_b W_{b\to a}(x/y)\, f_b(y,Q^2)\,. \qquad (1)$$

The anomalous dimensions (AD) $\gamma_{ab}(n)$ of the twist-2 Wilson operators $\mathcal{O}^a_{\mu_1,...,\mu_n}$ (hereafter $a_s = \alpha_s/(4\pi)$)

$$\gamma_{ab}(n) = \int_0^1 dx\; x^{n-1} W_{b\to a}(x) = \sum_{m=0}^{\infty} \gamma_{ab}^{(m)}(n) a_s^m,$$

All parton densities are multiplies by x, t.e.

structure function = combination of parton densities.

3. Method

(C.Lopez and F.J.Yndurain, 1980,1981), (A.V.K., 1994)

Here I present briefly the method, which leads to the possibility to replace the Mellin convolution of two functions

$$f_1(x) \otimes f_2(x) \equiv \int_x^1 \frac{dy}{y} f_1(y) f_2(x/y)$$

by a simple products at small x.

A. So, if $f_1(x) = B_k(x, Q^2)$ is perturbatively calculated Wilson kernel and $f_2(x) = x f_a(x, Q^2) \sim x^{-\delta}$ at $x \to 0$, then

$$f_1(x) \otimes f_2(x) \;\approx\; M_k(1+\delta, Q^2)\, f_2(x) \qquad (2)$$

where $M_k(1+\delta, Q^2)$ is the analytical continuation to non-integer arguments of the Mellin moment $M_k(n, Q^2)$ of $B_k(x, Q^2)$:

$$M_k(n, Q^2) \;=\; \int_0^1 x^{n-2} B_k(x, Q^2) \qquad (3)$$

The equation (2) is correct if the moment $M_k(n, Q^2)$ has no singularity at $n \to 1$.

B. The general case

$(M(n)$ **contains the singularity at** $n \to 1)$:

the form of subasymptotics of $f_2(x)$ starts to be important.

Let PD have the different forms:

- Regge-like form $xf_R(x) = x^{-\delta}\tilde{f}(x)$,
- Logarithmic-like form $xf_L(x) = x^{-\delta}ln(1/x)\tilde{f}(x)$,
- Bessel-like form $xf_I(x) = x^{-\delta}I_k(2\sqrt{\hat{d}ln(1/x)})\tilde{f}(x)$,

where $\tilde{f}(x)$ and its derivative $\tilde{f}'(x) \equiv d\tilde{f}(x)/dx$ are smooth at $x = 0$ and both are equal to zero at $x = 1$:

$$\tilde{f}(1) \; = \; \tilde{f}'(1) \; = \; 0$$

Then $(i = R, L, I)$

$$f_1(x) \otimes f_2(x) \;\approx\; \tilde{M}_k(1+\delta_i, Q^2)\, f_2(x),$$

where $\tilde{M}_{1+\delta_i} = M_{1+\delta}$ with $1/\delta \to 1/\tilde{\delta}_i$.

Regge-like behavior:

$$1/\tilde{\delta}_R \;=\; 1/\delta \left[1 - x^{\delta}\, \frac{\Gamma(1-\delta)\Gamma(\nu)}{\Gamma(1+\nu-\delta)}\right],$$

where $x f_R(x) \sim (1-x)^{\nu}$ at $x \to 1$.

The second term comes from low part of convolution integral

$$f_1(x) \otimes f_2(x) \;\equiv\; \int_x^1 \frac{dy}{y}\, f_1(x/y) f_2(y) \tag{4}$$

So,

$$\frac{1}{\tilde{\delta}_R} = \frac{1}{\delta} \quad \text{if} \quad x^\delta << 1$$

and

$$\frac{1}{\tilde{\delta}_R} = \ln\frac{1}{x} - [\Psi(1+\nu) - \Psi(1)] \quad \text{if} \quad \delta = 0$$

Analogously, for nonRegge behavior at $\delta \to 0$

$$\frac{1}{\tilde{\delta}_L} = \frac{1}{2}\ln\frac{1}{x} + O(1/ln(1/x)),$$

$$\frac{1}{\tilde{\delta}_I} = \sqrt{\frac{ln(1/x)}{\hat{d}}}\,\frac{I_{k+1}(2\sqrt{\hat{d}ln(1/x)})}{I_k(2\sqrt{\hat{d}ln(1/x)})}$$

3. Generalized double-logarithmic approach

(A.V.K. and G.Parente, 1998),
(A.Yu.Illarionov, A.V.K. and G.Parente, 2004)

(Generalized double asymptotic scaling)

1 Leading order without quarks (a pedagogical example)

At the momentum space, the solution of the DGLAP equation is

$$M_g(n,Q^2) \; = \; M_g(n,Q_0^2)e^{-d_{gg}(n)s},$$

where $M_g(n,Q^2)$ are the moments of the gluon distribution,

$$s = ln\left(\frac{a_s(Q_0^2)}{a_s(Q^2)}\right), \quad a_s(Q^2) = \frac{\alpha_s(Q^2)}{4\pi} \quad \text{and} \quad d_{gg} = \frac{\gamma_{gg}^{(0)}(n)}{2\beta_0}$$

The terms $\gamma_{gg}^{(0)}(n)$ and β_0 are respectively the LO coefficients of the gluon-gluon AD and the QCD β-function.

For any perturbatively calculable variable $Q(n)$, it is very convenient to separate the singular part when $n \to 1$ (denoted by "$\widehat{Q}$") and the regular part (marked as "$\overline{Q}$"):

$$Q(n) \;=\; \frac{\widehat{Q}}{n-1} + \overline{Q}(n)$$

Then, the above equation can be represented by the form

$$M_g(n,Q^2) \;=\; M_g(n,Q_0^2) e^{-\hat{d}_{gg} s_{LO}/(n-1)} e^{-\overline{d}_{gg}(n) s_{LO}},$$

with $\hat{\gamma}_{gg} = -8C_A$ and $C_A = N$ for $SU(N)$ group.

Finally, if one takes the flat boundary conditions
(i.e. min information about initial conditions, or
min contribution from initial conditions.)

$$xf_a(x,Q_0^2) \;=\; A_a, \quad \rightarrow \quad M_a(n,Q_0^2) \;=\; \frac{A_a}{n-1} \qquad (5)$$

(A.D.Rujula, S.L.Glashow, H.D.Politzer, S.B.Treiman, F.Wilczek and A.Zee, 1974)

Then, expanding the second exponential in the above equation

$$M_g^{cdl}(n, Q^2) \;=\; A_g \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\left(-\hat{d}_{gg} s_{LO}\right)^k}{(n-1)^{k+1}}$$

and using the Mellin transformation for $\left(ln(1/x)\right)^k$:

$$\int_0^1 dx x^{n-2} \left(ln(1/x)\right)^k \;=\; \frac{k!}{(n-1)^{k+1}}$$

we immediately obtain the well known double-logarithmic behavior

$$f_g^{cdl}(x, Q^2) \;=\; A_g \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(-\hat{d}_{gg} s_{LO}\right)^k \left(ln(1/x)\right)^k \;=\; A_g I_0(\sigma_{LO}),$$

where $I_0(\sigma_{LO})$ is the modified Bessel function with argument $\sigma_{LO} = 2\sqrt{\hat{d}_{gg} s_{LO} ln(x)}$. (R.D.Ball and S.Forte, 1994),

1.2 The more general case

For a regular kernel $\tilde{K}(x)$, having Mellin moment (nonsingular at $n \to 1$)

$$K(n) \;=\; \int_0^1 dx x^{n-2} \tilde{K}(x)$$

and the PD $f_a(x)$ in the form $I_\nu(\sqrt{\hat{d} ln(1/x)})$ we have the following equation

$$\tilde{K}(x) \otimes f_a(x) \;=\; K(1) f_a(x) + O(\sqrt{\frac{\hat{d}}{ln(1/x)}})$$

So, one can find the general solution for the LO gluon density without the influence of quarks

$$f_g(x,Q^2) \;=\; A_g I_0(\sigma_{LO}) e^{-\overline{d}_{gg}(1) s_{LO}} \;+\; O(\rho_{LO}),$$

where (R.D.Ball and S.Forte, 1994)

$$\rho_{LO} = \sqrt{\frac{\hat{d}_{gg} s_{LO}}{ln(x)}} = \frac{\sigma_{LO}}{2ln(1/x)}\,, \quad \overline{\gamma}_{gg}^{(0)}(1) = 22 + \frac{4}{3}f$$

and

$$\overline{d}_{gg}(1) = 1 + \frac{4f}{3\beta_0}$$

with f as the number of active quarks.

At the momentum space, the solution of the DGLAP equation at LO has the form (*after diagonalization*)

$$M_a(n,Q^2) = M_a^+(n,Q^2) + M_a^-(n,Q^2) \text{ and}$$
$$M_a^\pm(n,Q^2) = M_a^\pm(n,Q_0^2)e^{-d_\pm(n)s} = M_a^\pm e^{-\hat{d}_\pm s/(n-1)} e^{-\overline{d}_\pm(n)s},$$

where

$$M_a^\pm(n,Q^2) = \varepsilon_{ab}^\pm(n) M_b(n,Q^2), \qquad d_{ab} = \frac{\gamma_{ab}^{(0)}(n)}{2\beta_0},$$
$$d_\pm(n) = \frac{1}{2}[(d_{gg}(n)+d_{qq}(n))$$
$$\pm (d_{gg}(n)-d_{qq}(n))\sqrt{1+\frac{4d_{qg}(n)d_{gq}(n)}{(d_{gg}(n)-d_{qq}(n))^2}}]$$
$$\varepsilon_{qq}^\pm(n) = \varepsilon_{gg}^\mp(n) = \frac{1}{2}(1+\frac{d_{qq}(n)-d_{gg}(n)}{d_\pm(n)-d_\mp(n)}),$$

$$\varepsilon_{ab}^{\pm}(n) = \frac{d_{ab}(n)}{d_{\pm}(n) - d_{\mp}(n)}(a \neq b)$$

As the singular (when $n \to 1$) part of the $+$ component of the anomalous dimension is !!! $\hat{d}_{+} = \hat{d}_{gg} = -4C_A/\beta_0$!!! while the $-$ component does not exist: !!! $(\hat{d}_{-} = 0)$!!! , we consider below both cases separately.

2.1 The "+" component

The analysis of the "+" component is practically identical to the case studied before. The only difference lies in the appearance of new terms $\varepsilon^+_{ab}(n)$!!! . If they are expanded in the vicinity of $n = 1$ in the form $\varepsilon^+_{ab}(n) = \overline{\varepsilon}^+_{ab} + (n-1)\tilde{\varepsilon}^+_{ab}$, !!! then for the terms $\overline{\varepsilon}^+_{ab}$ multiplying $M_b(n, Q^2)$, we have the same results as in previous section:

$$\overline{\varepsilon}^+_{ab} M_b(n, Q^2) \stackrel{\mathcal{M}^{-1}}{\longrightarrow} \overline{\varepsilon}^+_{ab} A_b I_0(\sigma_{LO}) e^{-\overline{d}_+(1) s_{LO}} + O(\rho_{LO}),$$

where the symbol $\stackrel{\mathcal{M}^{-1}}{\longrightarrow}$ denotes the inverse Mellin transformation. The values of σ and ρ coincide with those defined in the previous section because $\hat{d}_+ = \hat{d}_{gg}$.

The terms $\tilde{\varepsilon}^{+}_{ab}$ that come with the additional factor $(n-1)$ in front, lead to the following results

$$(n-1)\tilde{\varepsilon}^{+}_{ab}\frac{A_b}{(n-1)}e^{-\hat{d}_+ s_{LO}/(n-1)} = \tilde{\varepsilon}^{+}_{ab}A_b \sum_{k=0}^{\infty}\frac{1}{k!}\frac{\left(-\hat{d}_+ s_{LO}\right)^k}{(n-1)^k}$$

$$\xrightarrow{\mathcal{M}^{-1}} \;\; \tilde{\varepsilon}^{+}_{ab}A_b \sum_{k=0}^{\infty}\frac{1}{k!}\frac{1}{(k-1)!}\left(-\hat{d}_+ s_{LO}\right)^k (ln(1/x))^{k-1}$$

$$= \;\; \tilde{\varepsilon}^{+}_{ab}A_b \rho_{LO} I_1(\sigma_{LO}),$$

i.e. the additional factor $(n-1)$ in momentum space leads to replacing the Bessel function $I_0(\sigma_{LO})$ by $\rho_{LO}I_1(\sigma_{LO})$ in x-space.

Thus, we obtain that the term $\varepsilon^{+}_{ab}(n)M_b(n,Q^2)$ leads to the following contribution in x space !!! :

$$(\overline{\varepsilon}^{+}_{ab}I_0(\sigma_{LO}) + \tilde{\varepsilon}^{+}_{ab}\rho_{LO}I_1(\sigma_{LO}))A_b e^{-\overline{d}_+(1)s_{LO}} \; + \; O(\rho_{LO})$$

Because the Bessel function $I_\nu(\sigma)$ has the ν-independent asymptotic behavior !!! $e^\sigma/\sqrt{\sigma}$ at $\sigma \to \infty$ (i.e. $x \to 0$), the second term is $O(\rho)$ and must be kept only !!! when $\overline{\varepsilon}^+_{ab} = 0$. This is the case for the quark distribution at the LO approximation.

Using the concrete AD values, one has

$$f_g^+(x,Q^2) = (A_g + \frac{4}{9}A_q) I_0(\sigma_{LO}) e^{-\overline{d}_+(1)s_{LO}} + O(\rho_{LO}) \quad \text{and}$$

$$f_q^+(x,Q^2) = \frac{f}{9}(A_g + \frac{4}{9}A_q) \rho_{LO} I_1(\sigma_{LO}) e^{-\overline{d}_+(1)s_{LO}} + O(\rho_{LO})$$

where $\overline{d}_+(1) = 1 + 20f/(27\beta_0)$.

2.2 the "–" component

In this case the anomalous dimension is regular !!! and one has

$$\varepsilon^-_{ab}(n)A_b e^{-d_-(n)s} \xrightarrow{\mathcal{M}^{-1}} \varepsilon^-_{ab}(1)A_b e^{-d_-(1)s_{LO}} + O(x)$$

Using the concrete AD values !!! , we have

$$f_g^-(x,Q^2) = -\frac{4}{9}A_q e^{-d_-(1)s_{LO}} + O(x) \text{ and}$$
$$f_q^-(x,Q^2) = A_q e^{-d_-(1)s_{LO}} + O(x),$$

where $d_-(1) = 16f/(27\beta_0)$.

Finally we present the full small x asymptotic results for PD and F_2 structure function at LO of perturbation theory:

$$f_a(x,Q^2) = f_a^+(x,Q^2) + f_a^-(x,Q^2) \quad \text{and}$$
$$F_2(x,Q^2) = e \cdot f_q(z,Q^2)$$

where f_q^+,f_g^+, f_q^- and f_g^- were already given before and $e = \Sigma_1^f e_i^2/f$ is the average charge square of the f active quarks.

Extension to NLO is trivial and can be found in (A.V.K. and G.Parente, 1998)

4. Fits of HERA data

At low x, the structure function $F_2(x, Q^2)$ is related to parton densities as (A.V.K. and G.Parente, 1998)

at LO

$$F_2(x, Q^2) = \frac{5}{18} f_q(x, Q^2)$$

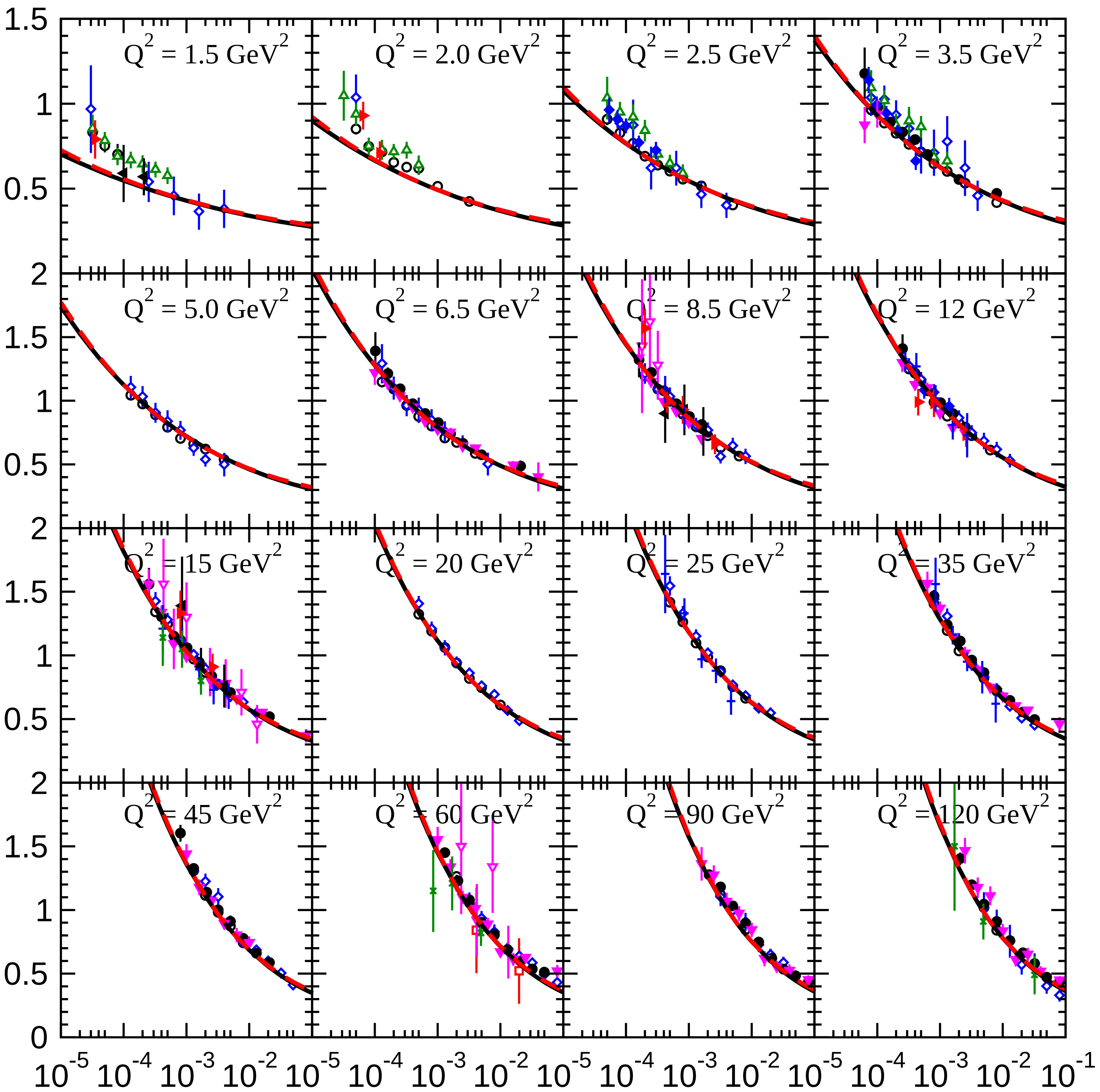
at NLO

$$F_2(x, Q^2) = \frac{5}{18} \left[f_q(x, Q^2) + \frac{2f}{3} a_s(Q^2) f_g(x, Q^2) \right].$$

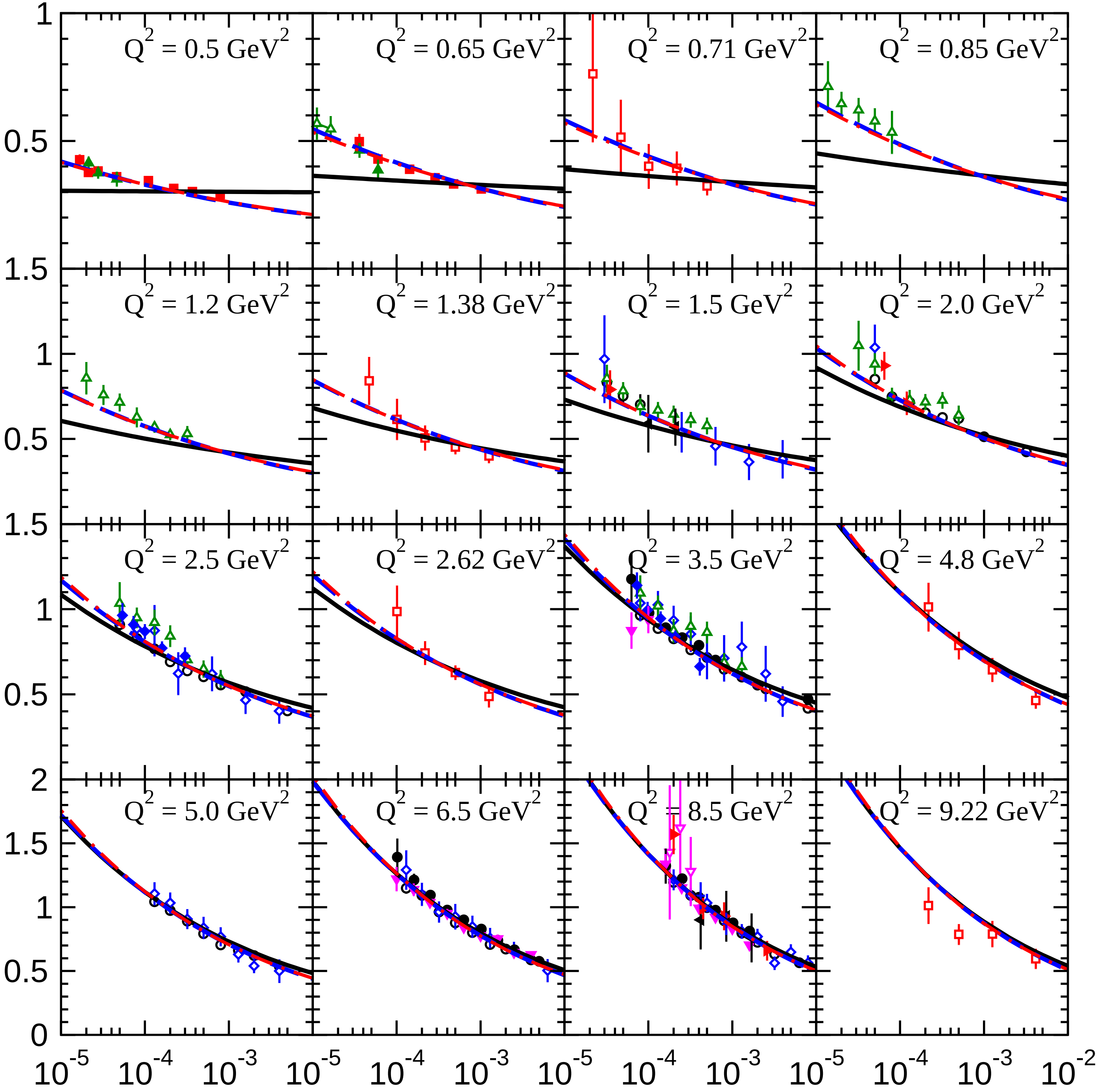
Fits of HERA experimental data of the structure function $F_2(x, Q^2)$ (A.Yu.Illarionov, A.V.K. and G.Parente, 2004)

!!! Only three parameters: Q_0^2, A_q and A_g

Λ_{QCD} **cannon be extract in small** x **Physics.**



Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 12 GeV^2
Q^2 = 15 GeV^2
Q^2 = 20 GeV^2
Q^2 = 25 GeV^2
Q^2 = 35 GeV^2
Q^2 = 45 GeV^2
Q^2 = 60 GeV^2
Q^2 = 90 GeV^2
Q^2 = 120 GeV^2



Q^2 = 0.5 GeV^2
Q^2 = 0.65 GeV^2
Q^2 = 0.71 GeV^2
Q^2 = 0.85 GeV^2
Q^2 = 1.2 GeV^2
Q^2 = 1.38 GeV^2
Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 2.62 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 4.8 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 9.22 GeV^2

5. Analytical and "frozen" coupling constants

Two modifications of the coupling constant (G.Cvetic, A.Yu.Illarinov, B.A. Kniehl, and A.V.K., 2009); (A.V.K. and B.G. Shaikhatdenov, 2012)

A. More phenomenological.

(G.Curci, M.Greco and Y.Sristava, 1979), (M.Greco, G. Penso and Y.Sristava, 1980), (N.N.Nikolaev and B.M.Zakharov, 1991,1992), (B.Badelek,J.Kwiecinski and A.Stasto, 1997), (A.M.Badalian and Yu.A.Simonov, 1997)

We introduce freezing of the coupling constant by changing its argument $Q^2 \to Q^2 + M_\rho^2$, where M_ρ is sually the ρ-meson mass. Thus, in the formulae of the previous Sections we should do the following replacement

$$a_s(Q^2) \to a_{fr}(Q^2) \equiv a_s(Q^2 + M_\rho^2) \tag{6}$$

B. Theoretical approach.

Incorporates the Shirkov-Solovtsov idea (D.V.Shirkov and L.I.Solovtsov, 1997), about analyticity of the coupling constant that leads to the additional its power dependence.

(K.A.Milton, A.V. Nesterenko, O.Solovtsova, G. Cvetic, C. Valenzuela, I. Schmidt, O. Teryaev, N. Stefanis, A. Bakulev, S. Mikhailov, ...)

Then, in the formulae of the previous Section the coupling constant $a_s(Q^2)$ should be replaced as follows

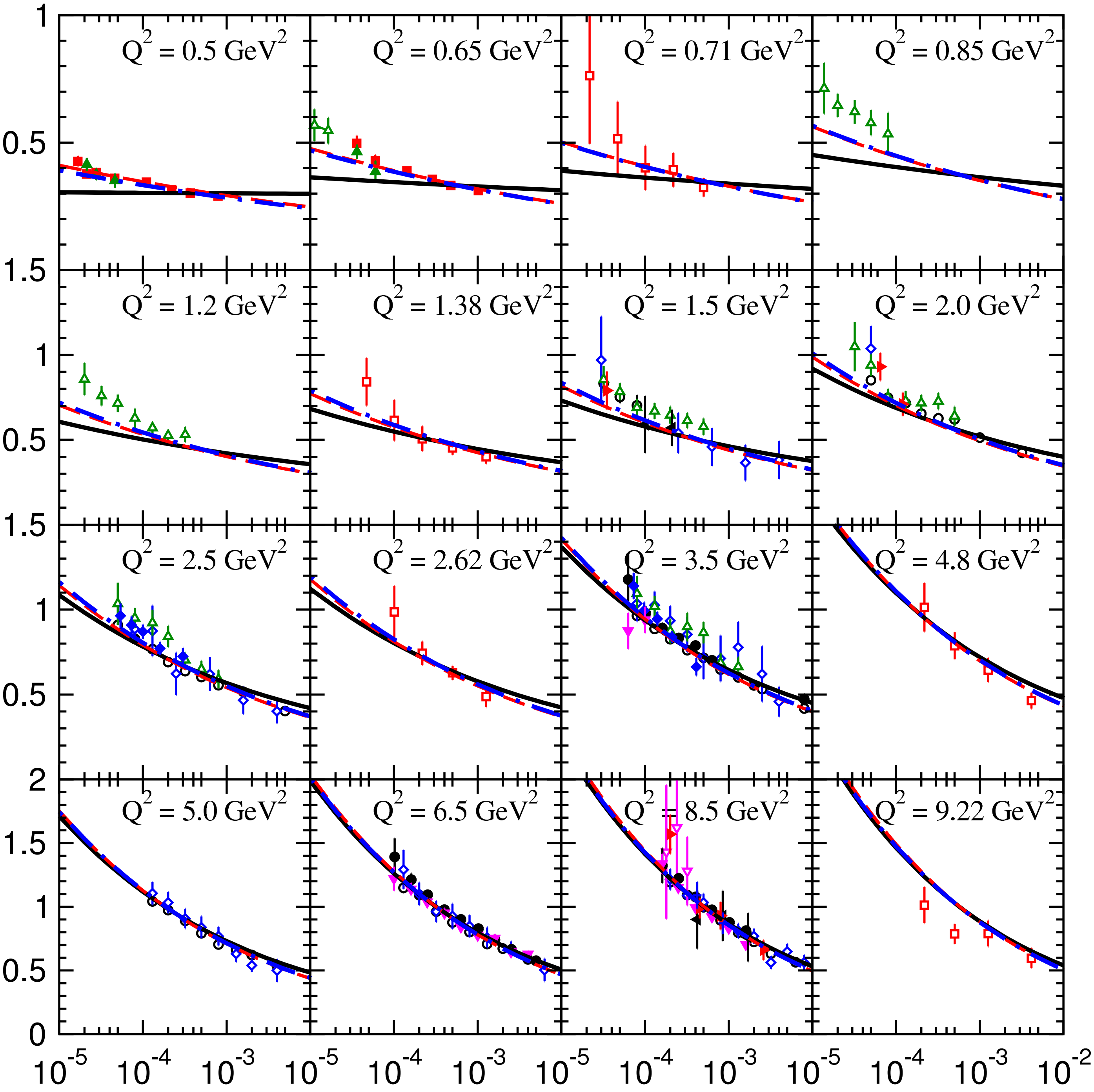
$$a_{an}^{LO}(Q^2) = a_s(Q^2) - \frac{1}{\beta_0} \frac{\Lambda_{LO}^2}{Q^2 - \Lambda_{LO}^2} \quad (7)$$

at the LO approximation and

$$a_{an}(Q^2) = a_s(Q^2) - \frac{1}{2\beta_0} \frac{\Lambda^2}{Q^2 - \Lambda^2} + ... \quad (8)$$

at the NLO approximation, where the symbol ... marks numerically small terms.

The replacement (7) and (8) is applicable only for ruther large values of Q^2!!!
For lower Q^2 values it is better to use the fraction analytic perturbation theory (A. Bakulev, S. Mikhailov, N. Stefanis, 2005) , but it direct application is ruther difficult.



Q^2 = 0.5 GeV^2
Q^2 = 0.65 GeV^2
Q^2 = 0.71 GeV^2
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Q^2 = 1.38 GeV^2
Q^2 = 1.5 GeV^2
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Q^2 = 2.5 GeV^2
Q^2 = 2.62 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 4.8 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 9.22 GeV^2

Table 1: The result of the LO and NLO fits to H1 and ZEUS data for different low Q^2 cuts. In the fits f is fixed to 4 flavors.

	A_g	A_q	Q_0^2 [GeV^2]	$\chi^2/n.o.p.$
$Q^2 \geq 1.5\text{GeV}^2$				
LO	0.784±.016	0.801±.019	0.304±.003	754/609
LO&an.	0.932±.017	0.707±.020	0.339±.003	632/609
LO&fr.	1.022±.018	0.650±.020	0.356±.003	547/609
NLO	-0.200±.011	0.903±.021	0.495±.006	798/609
NLO&an.	0.310±.013	0.640±.022	0.702±.008	655/609
NLO&fr.	0.180±.012	0.780±.022	0.661±.007	669/609
$Q^2 \geq 0.5\text{GeV}^2$				
LO	0.641±.010	0.937±.012	0.295±.003	1090/662
LO&an.	0.846±.010	0.771±.013	0.328±.003	803/662
LO&fr.	1.127±.011	0.534±.015	0.358±.003	679/662
NLO	-0.192±.006	1.087±.012	0.478±.006	1229/662
NLO&an.	0.281±.008	0.634±.016	0.680±.007	633/662
NLO&fr.	0.205±.007	0.650±.016	0.589±.006	670/662

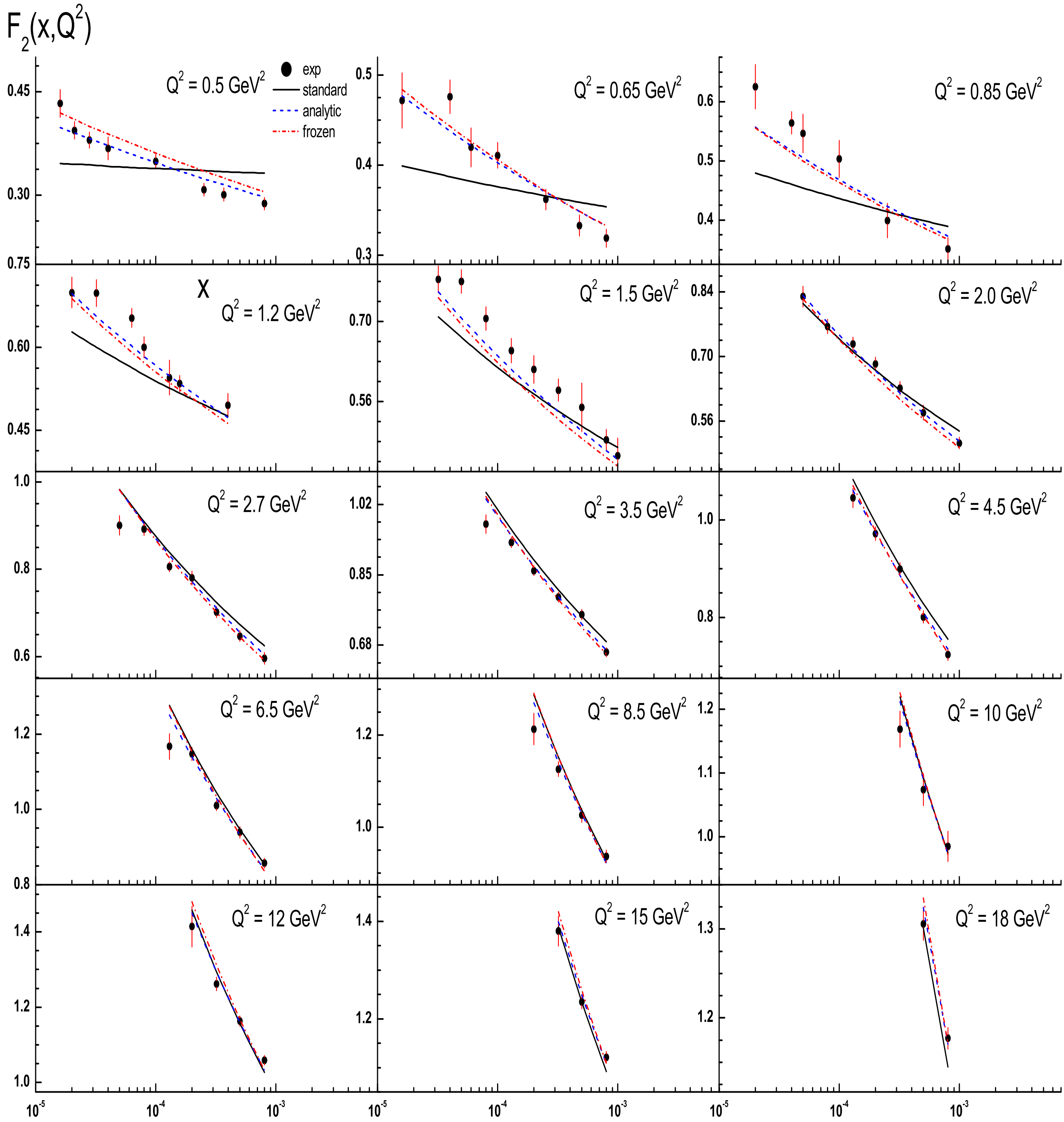
- Usage of the analytical and "frozen" coupling constants leads to improvement with data: χ^2 decreased twicely
- Really, no difference between results based on the analytical and "frozen" coupling constants.

!!! One example of application the analytical and "frozen" coupling constants: (A.V.Kotikov, A.V.Lipatov and N.P.Zotov, 2004)

New $H1\&ZEUS$ (2010) experimental data for F_2:
(F.D. Aaron *et al.*, 2010)
there is a good agreement for $Q^2 \geq 0.5$ GeV^2.

Table 2: The results of LO and NLO fits to H1 & ZEUS data with various lower cuts on Q^2; in the fits the n
f is fixed to 4.

	A_g	A_q	Q_0^2 [GeV^2]	$\chi^2/n.d.f.$
$Q^2 \geq 5\text{GeV}^2$				
LO	0.623±0.055	1.204±0.093	0.437±0.022	1.00
LO&an.	0.796±0.059	1.103±0.095	0.494±0.024	0.85
LO&fr.	0.782±0.058	1.110±0.094	0.485±0.024	0.82
NLO	-0.252±0.041	1.335±0.100	0.700±0.044	1.05
NLO&an.	0.102±0.046	1.029±0.106	1.017±0.060	0.74
NLO&fr.	-0.132±0.043	1.219±0.102	0.793±0.049	0.86
$Q^2 \geq 3.5\text{GeV}^2$				
LO	0.542±0.028	1.089±0.055	0.369±0.011	1.73
LO&an.	0.758±0.031	0.962±0.056	0.433±0.013	1.32
LO&fr.	0.775±0.031	0.950±0.056	0.432±0.013	1.23
NLO	-0.310±0.021	1.246±0.058	0.556±0.023	1.82
NLO&an.	0.116±0.024	0.867±0.064	0.909±0.330	1.04
NLO&fr.	-0.135±0.022	1.067±0.061	0.678±0.026	1.27
$Q^2 \geq 2.5\text{GeV}^2$				
LO	0.526±0.023	1.049±0.045	0.352±0.009	1.87
LO&an.	0.761±0.025	0.919±0.046	0.422±0.010	1.38
LO&fr.	0.794±0.025	0.900±0.047	0.425±0.010	1.30
NLO	-0.322±0.017	1.212±0.048	0.517±0.018	2.00
NLO&an.	0.132±0.020	0.825±0.053	0.898±0.026	1.09
NLO&fr.	-0.123±0.018	1.016±0.051	0.658±0.021	1.31
$Q^2 \geq 0.5\text{GeV}^2$				
LO	0.366±0.011	1.052±0.016	0.295±0.005	5.74
LO&an.	0.665±0.012	0.804±0.019	0.356±0.006	3.13
LO&fr.	0.874±0.012	0.575±0.021	0.368±0.006	2.96
NLO	-0.443±0.008	1.260±0.012	0.387±0.010	6.62
NLO&an.	0.121±0.008	0.656±0.024	0.764±0.015	1.84
NLO&fr.	-0.071±0.007	0.712±0.023	0.529±0.011	2.79

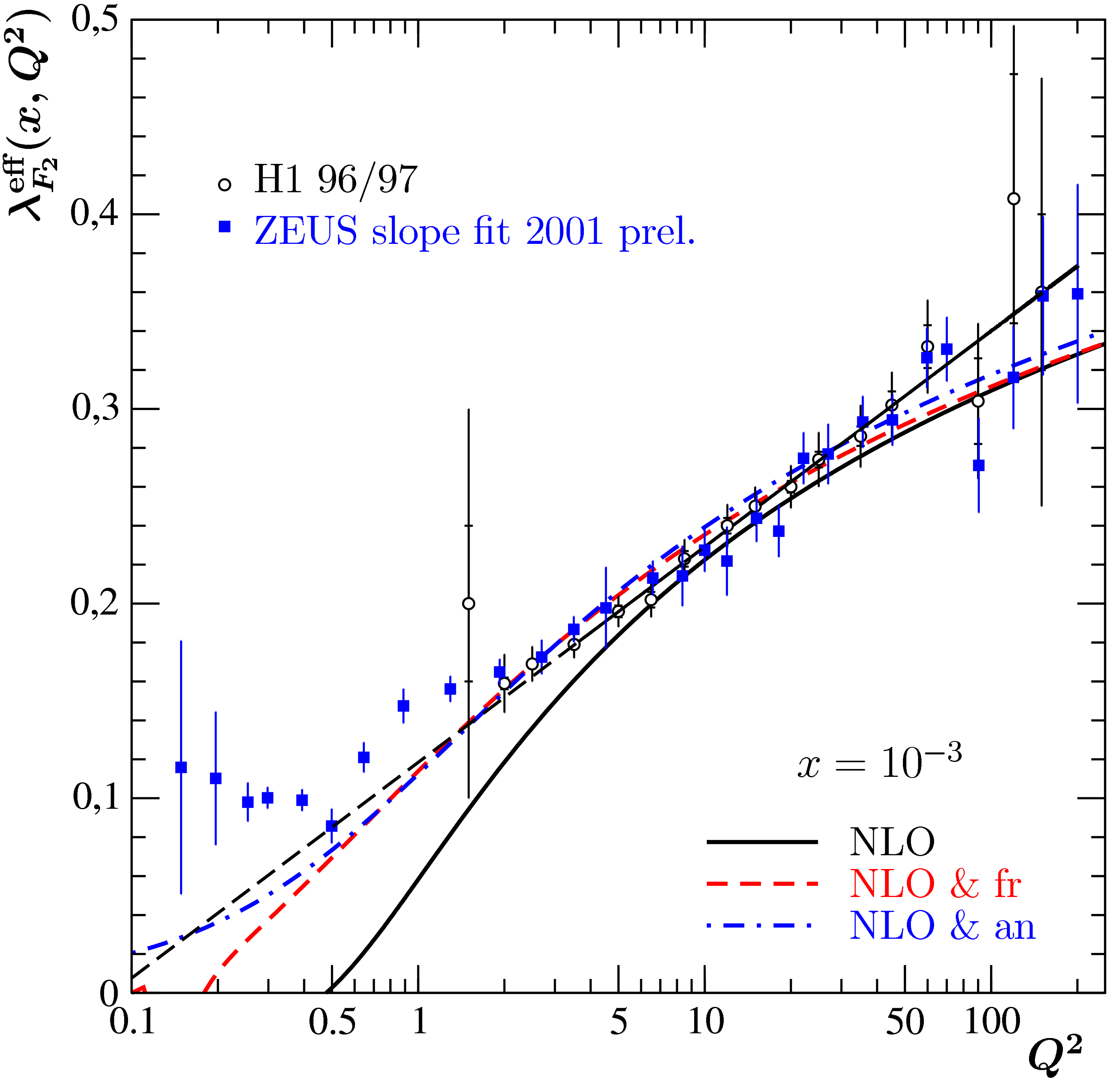


F2(x,Q2)
Q2 = 0.5 GeV2
exp
standard
analytic
frozen
Q2 = 0.65 GeV2
Q2 = 0.85 GeV2
Q2 = 1.2 GeV2
Q2 = 1.5 GeV2
Q2 = 2.0 GeV2
Q2 = 2.7 GeV2
Q2 = 3.5 GeV2
Q2 = 4.5 GeV2
Q2 = 6.5 GeV2
Q2 = 8.5 GeV2
Q2 = 10 GeV2
Q2 = 12 GeV2
Q2 = 15 GeV2
Q2 = 18 GeV2
X

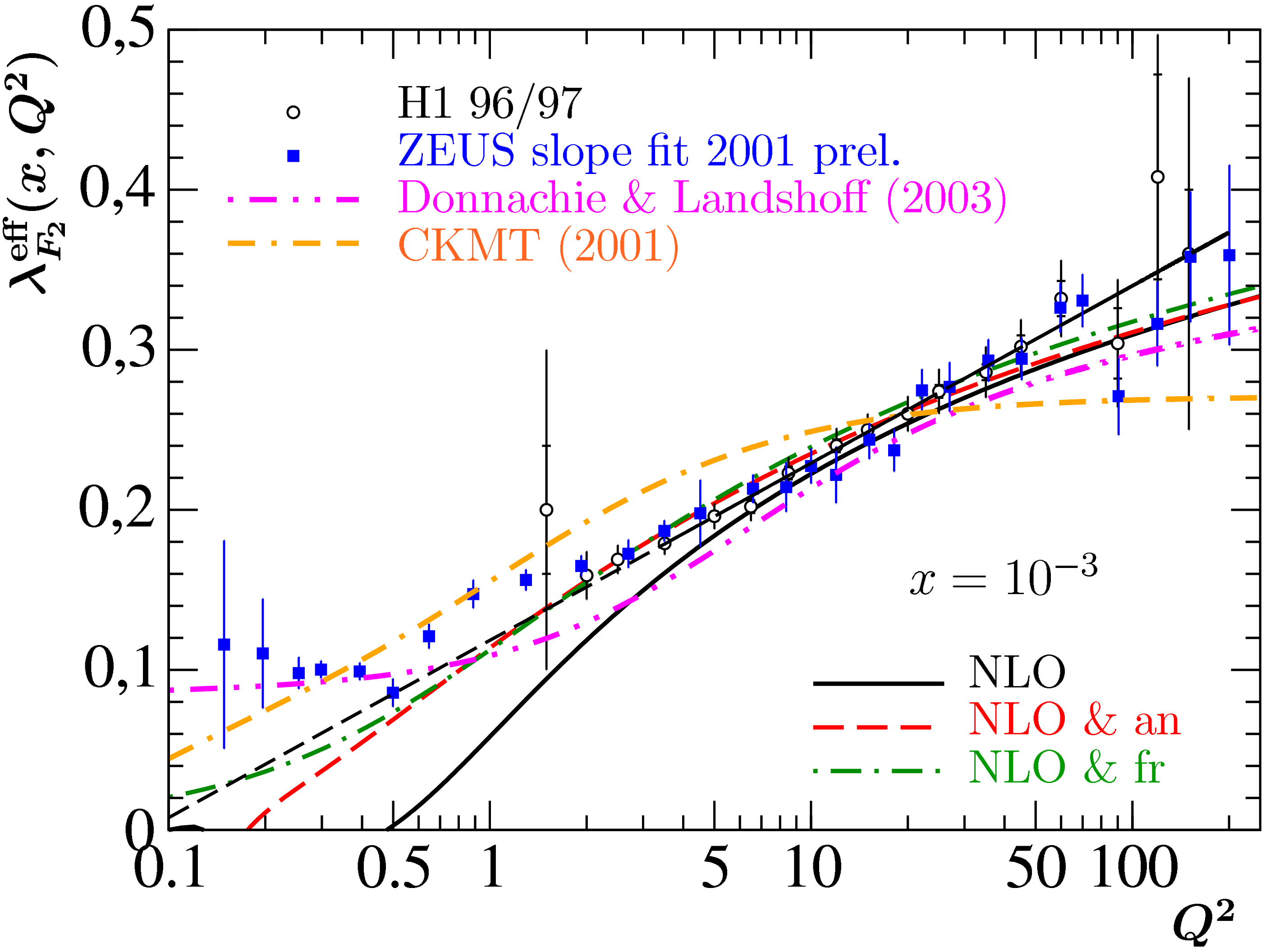
The results for F_2 and for the slope of the SF F_2

The double-logarithmic behaviour can mimic a power law shape over a limited region of x, Q^2.

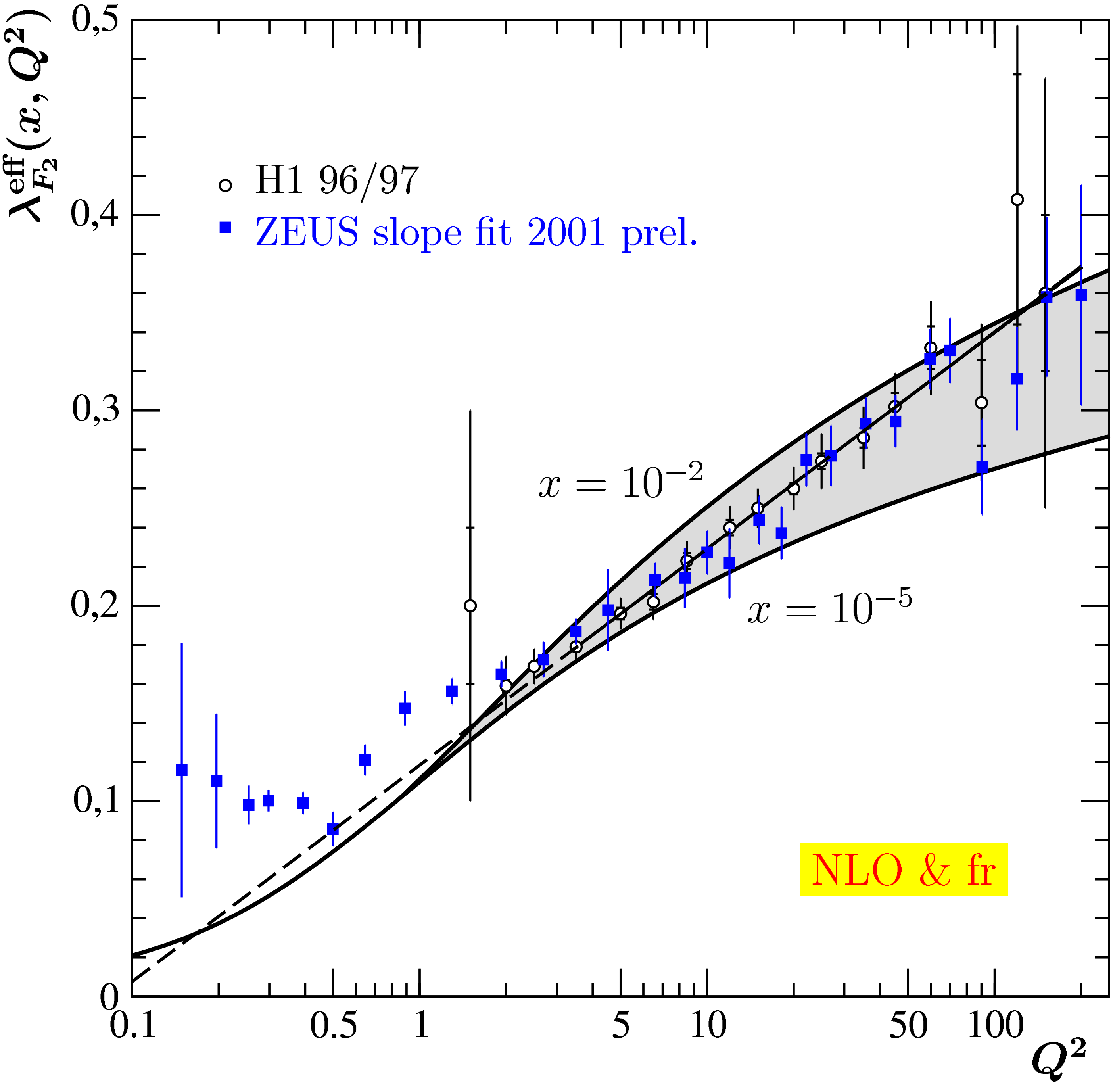
$$f_a(x,Q^2) \sim x^{-\lambda_a^{eff}(x,Q^2)} \quad \text{and} \quad F_2(x,Q^2) \sim x^{-\lambda_{F2}^{eff}(x,Q^2)}$$



$\lambda^{\rm eff}_{F_2}(x, Q^2)$
0,5
0,4
0,3
0,2
0,1
0
0.1
0.5
1
5
10
50
100
Q^2
H1 96/97
ZEUS slope fit 2001 prel.
$x = 10^{-3}$
NLO
NLO & fr
NLO & an



$\lambda^{\rm eff}_{F_2}(x,Q^2)$
0,5
0,4
0,3
0,2
0,1
0
H1 96/97
ZEUS slope fit 2001 prel.
Donnachie & Landshoff (2003)
CKMT (2001)
$x = 10^{-3}$
NLO
NLO & an
NLO & fr
0.1
0.5
1
5
10
50
100
Q^2



$\lambda^{\rm eff}_{F_2}(x,Q^2)$
0,5
0,4
0,3
0,2
0,1
0
H1 96/97
ZEUS slope fit 2001 prel.
$x = 10^{-2}$
$x = 10^{-5}$
NLO & fr
0.1
0.5
1
5
10
50
100
Q^2

6. Charm contribution

Consider the contribution of heavy quarks separately (really only charm one). So, at NLO ($f = 3$) in the framework of photon-gluon fusion (PGF)

$$\begin{aligned} F_2(x,Q^2) &= F_2^q(x,Q^2) + F_2^Q(x,Q^2), \\ F_2^q(x,Q^2) &= e_f \left[f_q(x,Q^2) + \frac{2f}{3} a_s(Q^2) f_g(x,Q^2) \right], \\ F_2^Q(x,Q^2) &\equiv F_2^c(x,Q^2) \approx M_{2,g}(1,Q^2,\mu^2) f_g(x,\mu^2), \end{aligned}$$

where $M_{2,g}(1,Q^2,\mu^2,m_c^2)$ is the first moment of the corresponding Wilson coefficient function $C_{2,g}(x,Q^2,\mu^2,m_c^2)$.

Through NLO, $M_{2,g}(1,Q^2,\mu^2,m_c^2)$ exhibits the structure

$$\begin{aligned} M_{2,g}(1,Q^2,\mu^2,m_c^2) &= e_i^2 a_s(\mu) \{ M_{2,g}^{(0)}(1,c) \\ &+ a_s(\mu) \left[M_{2,g}^{(1)}(1,c) + M_{2,g}^{(2)}(1,c) \ln \frac{\mu^2}{m_c^2} \right] \} + \mathcal{O}(a_s^3). \end{aligned}$$

The LO coefficient function of PGF can be obtained from the QED case (V.N. Baier, V.S. Fadin and V.A. Khoze, 1966), (V.G. Zima, 1972), (V.M. Budnev, I.F. Ginzburg, G.V. Meledin and V.G. Serbo, 1974) by adjusting coupling constants and color factors, and they read (E. Witten, 1975), (J.P. Leveille and T.J. Weile, 1977), (V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, 1978)

$$\begin{aligned} C_{2,g}^{(0)}(x,c) &= -2x\{[1-4x(2-c)(1-x)]\beta \\ &-[1-2x(1-2c)+2x^2(1-6c-4c^2)]L(\beta)\}, \end{aligned}$$

where

$$c = \frac{m_c^2}{Q^2}, \qquad \beta(x) = \sqrt{1-\frac{4cx}{1-x}}, \qquad L(\beta) = \ln\frac{1+\beta}{1-\beta}.$$

Performing the Mellin transformation

$$M_{2,g}^{(0)}(1,c) \;=\; \int_0^b dx x^{n-1} C_{2,g}^{(0)}(x,c),$$

we find

$$M_{2,g}^{(0)}(1,c) \;=\; \frac{2}{3}[1+2(1-c)J(c)]$$

with

$$J(c) = -\sqrt{b}\ln t, \qquad t = \frac{1-\sqrt{b}}{1+\sqrt{b}}, \qquad b = \frac{1}{1+4c}.$$

The NLO coefficient functions of PGF are rather lengthy and not published in print; they are only available as computer codes. (E. Laenen, S. Riemersma, J. Smith and W.L. van Neerve, 1992)

For $x \ll 1$, it is possible to use the compact form (S. Catani, M. Ciafaloni and F. Hautmann, 1994)

$$C_{2,g}^{(j)}(x,c) = \beta R_{2,g}^{(j)}(1,c),$$

with

$$\begin{aligned} R_{2,g}^{(1)}(1,c) &= \frac{8}{9} C_A [5 + (13-10c)J(c) + 6(1-c)I(c)], \\ R_{2,g}^{(2)}(1,c) &= -4 C_A M_{2,g}^{(0)}(1,c), \end{aligned}$$

where $C_A = N$ for the color gauge group SU(N), and

$$I(c) = -\sqrt{b}\left[\zeta(2) + \frac{1}{2}\ln^2 t - \ln(bc)\ln t + 2\,\mathrm{Li}_2(-t)\right],$$

with $\mathrm{Li}_2(x) = -\int_0^1 (dy/y)\ln(1-xy)$ is the dilogarithmic function.

The Mellin transforms of $C_{k,g}^{(j)}(x,c)$ exhibit singularities in the limit $n \to 1$. So, now we have the terms involving $1/\delta_\pm$, which correspond to singularities of the Mellin moments $M_{2,g}^\pm(n)$ at $n \to 1$ and depend on the exact form of the subasymptotic low-x behavior encoded in $f_g^\pm(x,\mu^2)$. The modification is simple:

$$\frac{1}{n-1} \to \frac{1}{\tilde{\delta}_\pm}, \qquad \frac{1}{\tilde{\delta}_\pm} = \frac{1}{f_g^\pm(\hat{x},\mu^2)} \int_{\hat{x}}^1 \frac{dy}{y} f_g^\pm(y,\mu^2),$$

where $\hat{x} = x/b$. In the considered case

$$\frac{1}{\tilde{\delta}_+} \approx \frac{1}{\rho(\hat{x})} \frac{I_1(\sigma(\hat{x}))}{I_0(\sigma(\hat{x}))}, \qquad \frac{1}{\tilde{\delta}_-} \approx \ln\frac{1}{\hat{x}},$$

where σ and ρ were introduced earlier.

Because the ratio $f_g^-(x,Q^2)/f_g^+(x,Q^2)$ is rather small at the Q^2 values considered, then

$$F_2^c(x,Q^2) \approx \tilde{M}_{2,g}(1,\mu^2,c)xf_g(x,\mu^2),$$

where $\tilde{M}_{2,g}(1,\mu^2)$ is obtained from $M_{2,g}(n,\mu^2)$ by taking the limit $n \to 1$ and replacing $1/(n-1) \to 1/\tilde{\delta}_+$:

$$M_{2,g}^{(j)}(1,c) \to \tilde{M}_{2,g}^{(j)}(1,c) \approx \left[\frac{1}{\delta_+} - \ln(bc) - \frac{J(c)}{b}\right] R_{2,g}^{(j)}(1,c) \quad (j=1,2)$$

with $R_{2,g}^{(j)}(1,a) \quad (j=1,2)$ are given above.

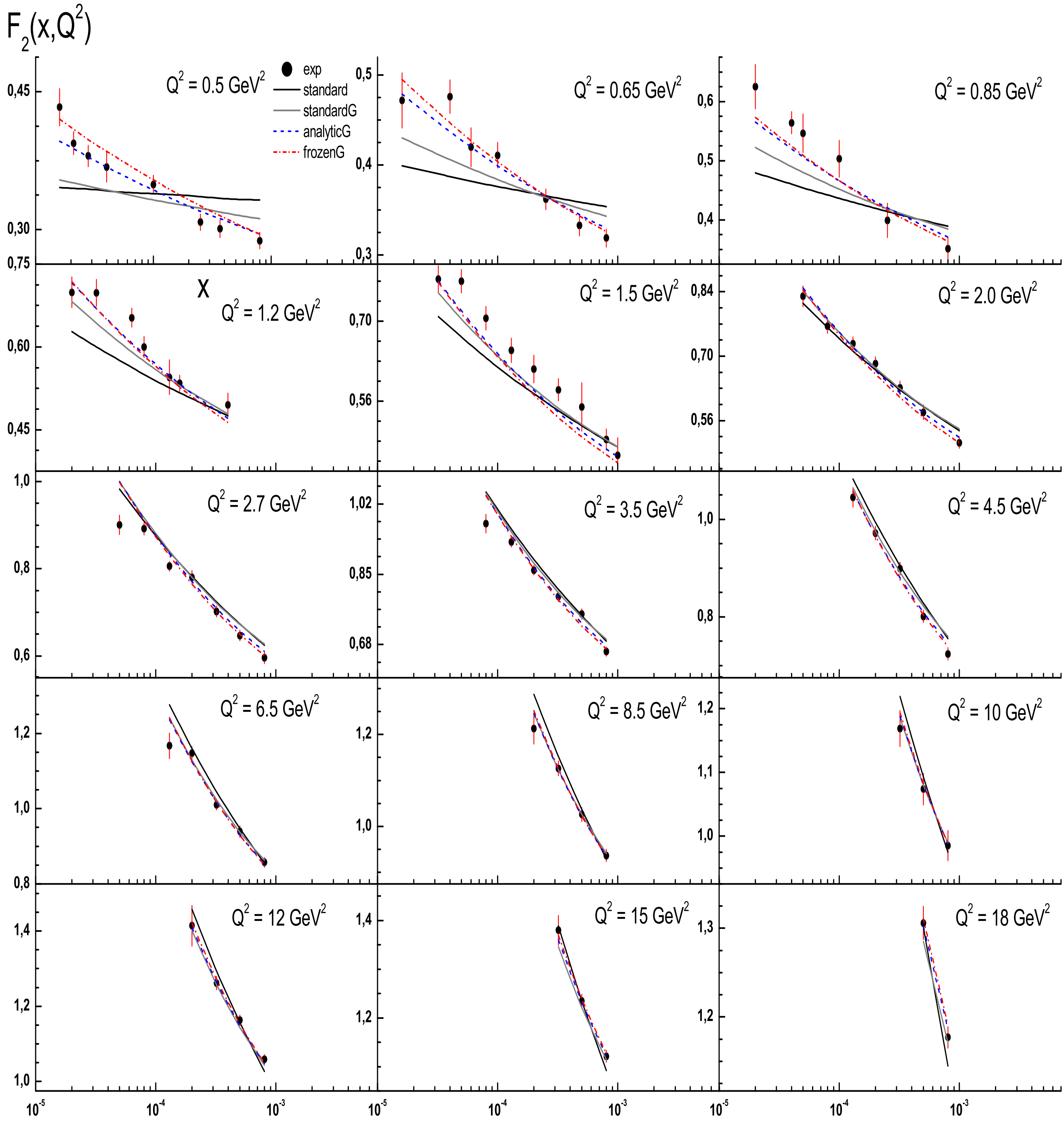
3 Results for F_2

Using above equations, we fit the new $H1\&ZEUS$ data for F_2. (F.D. Aaron *et al.*, 2010) using our formulas

$$
\begin{aligned}
F_2(x,Q^2) &= F_2^q(x,Q^2) + F_2^c(x,Q^2), \\
F_2^q(x,Q^2) &= \frac{2}{9}\left[f_q(x,Q^2) + 2a_s(Q^2) f_g(x,Q^2)\right], \\
F_2^c(x,Q^2) &= \frac{4}{9} a_s(Q^2) M_{2,g}^{(0)}(1,c) f_g(x,\mu^2), \\
M_{2,g}^{(0)}(1,c) &= \frac{2}{3}[1 + 2(1-c)J(c)]
\end{aligned}
\qquad (9)
$$

As for our input parameters, we choose $m_c = 1.25$ GeV in agreement with Particle Data Group.

The results are in ruther good agreement in the case of the analytic and “frozen” coupling constnats.



F2(x,Q2)
exp
standard
standardG
analyticG
frozenG
Q2 = 0.5 GeV2
Q2 = 0.65 GeV2
Q2 = 0.85 GeV2
Q2 = 1.2 GeV2
Q2 = 1.5 GeV2
Q2 = 2.0 GeV2
Q2 = 2.7 GeV2
Q2 = 3.5 GeV2
Q2 = 4.5 GeV2
Q2 = 6.5 GeV2
Q2 = 8.5 GeV2
Q2 = 10 GeV2
Q2 = 12 GeV2
Q2 = 15 GeV2
Q2 = 18 GeV2
X

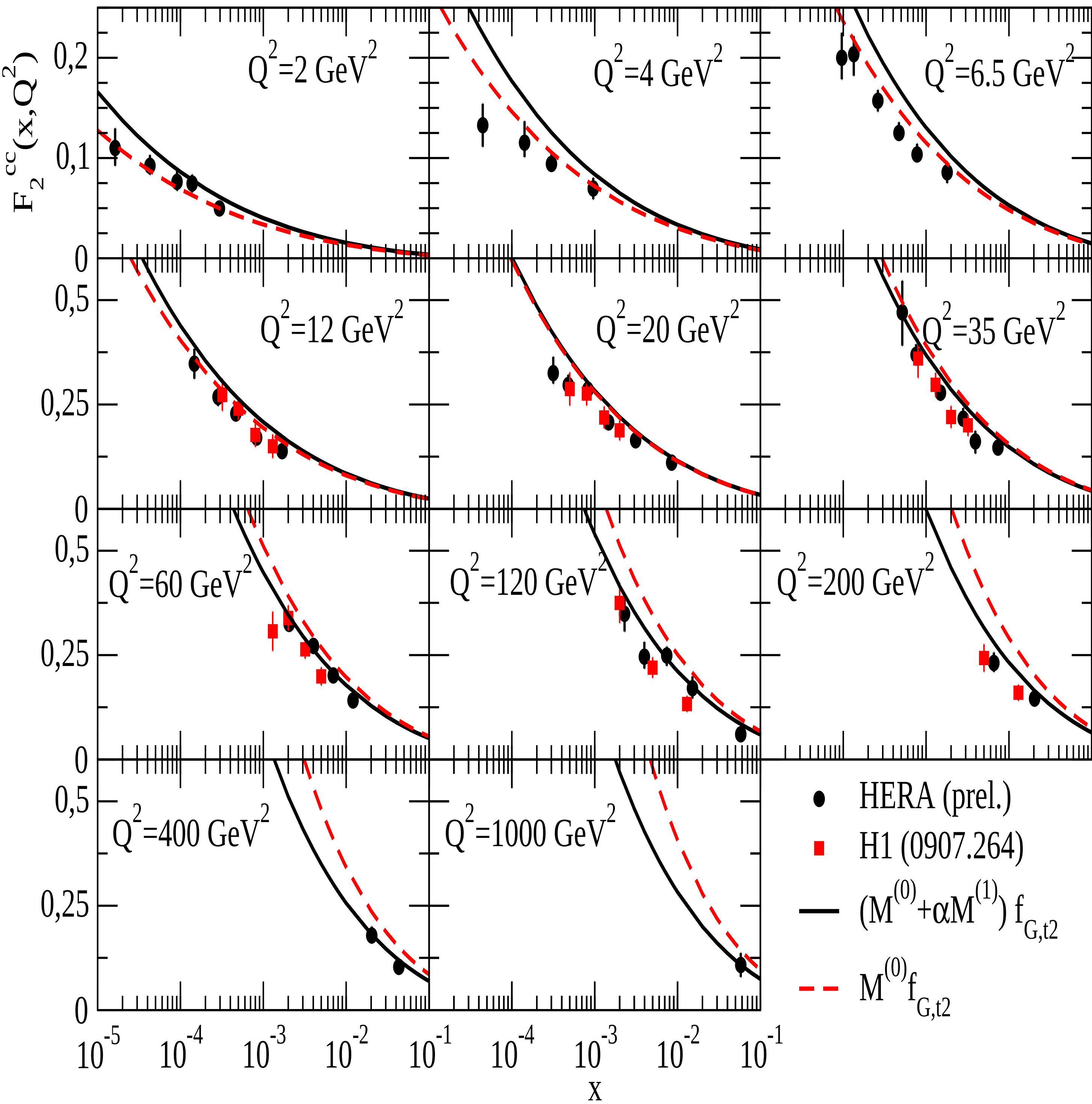
4 Results for F_2^c

We use the LO and NLO formulas

$$F^c_{2.LO}(x,Q^2) = \frac{4}{9}a_s(Q^2)M^{(0)}_{2,g}(1,c)f_g(x,\mu^2),$$
$$F^c_{2,NLO}(x,Q^2) = \frac{4}{9}a_s(Q^2)\{M^{(0)}_{2,g}(1,c)$$
$$+a_s(\mu)\left[M^{(1)}_{2,g}(1,c) + M^{(2)}_{2,g}(1,c)\ln\frac{\mu^2}{m_c^2}.\right]\}$$

In the NLO case we put $\mu^2 = Q^2 + 4m_c^2$, which is the standard scale in heavy quark production.

The PDF parameters μ_0^2, A_q and A_g have been fixed in the fits of F_2 experimental data (of $H1$ Collaboration). So, the analysis of F_2^c of $H1$ Collaboration has no free parameters. We have got a ruther good agreement with H1 experimental data.



$F_2^{cc}(x,Q^2)$
Q^2=2 GeV^2
Q^2=4 GeV^2
Q^2=6.5 GeV^2
Q^2=12 GeV^2
Q^2=20 GeV^2
Q^2=35 GeV^2
Q^2=60 GeV^2
Q^2=120 GeV^2
Q^2=200 GeV^2
Q^2=400 GeV^2
Q^2=1000 GeV^2
0
0,1
0,2
0,25
0,5
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x
HERA (prel.)
H1 (0907.264)
$(M^{(0)}+\alpha M^{(1)})\, f_{G,t2}$
$M^{(0)} f_{G,t2}$

As a next step. we will study an agreement with the new $H1\&ZEUS$ data for F_2^c (H. Abramowitz *et al.*, 2012) using the corresponding analysis of $H1\&ZEUS$ data for F_2 (F.D. Aaron *et al.*, 2010)

Conclusion

- I have demonstrated the low x asymptotics of parton densities and SF F_2 and F_2^c.
- Low x asymptotics of F_2 are in good agreement with data from HERA at $Q^2 \geq 2.5$ GeV^2.
- Usage of the analytical and "frozen" coupling constants leads to improvement with data from HERA at $Q^2 \leq 2.5$ GeV^2, including the new H1+ZEUS data for F_2.
  (F.D. Aaron *et al.*, 2010).

Next steps:

- To add the NNLO corrections (which has $\sim 1/(n-1)^2$ poles at $n \to 1$). So, the NNLO small-x asyptotics $\sim exp[\sim (\ln(1/x))^{2/3}]$ is more singular then the corresponding LO and NLO ones $\sim exp[\sim \sqrt{\ln(1/x)}]$.
- To consider the new H1+ZEUS data for F_2^c.
  (H. Abramowitz *et al.*, 2012).