Multiplicity and membranes collision in modified AdS

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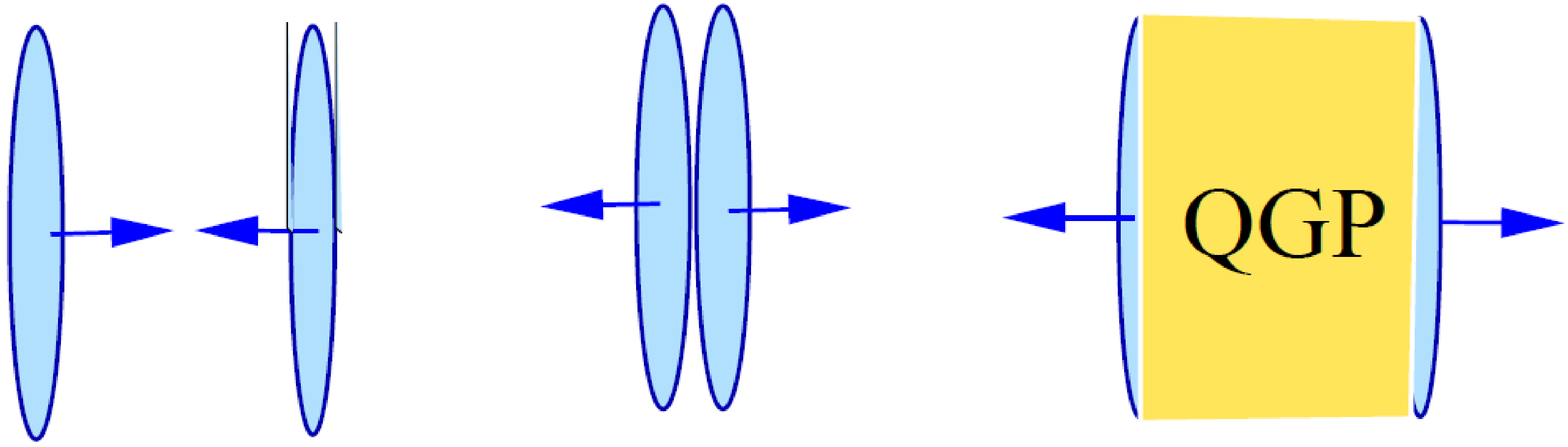
QGP

Quark Gluon Plasma (QGP) has been discovered in Au+Au collision at energy 100 GeV for nucleon in 2005 @ RHIC

QGP formation



5D gravity and 4D field theory are related

In holographic approch clasical gravity in AdS_5 discribes strong coupling field theory in 4D Minkowski space

There is hypothesis that QGP formation in 4D space corresponds to Black Holes creation in dual 5D space.

Gubser, Klebanov, Polyakov, 9802109

Witten, 9802150

The gravitational shock wave in AdS_5 space is dual to ultrarelativistic heavy-ion in 4D space-time.

Thus,

- heavy-ion collisions can be represented such as gravitational shock waves collisions in AdS_5

- QGP formation is equivalent BH creation in AdS_5

Gubser et al.; 0805.1551, 0902.4062

Problem:

- How to get experimental dependence of multiplicity on energy from holographic model.

- Simplest holographic model is related with use of $\mathcal{N}=4$ SYM

  [But QCD is not SYM]

- Our goal: to study more complicate models to fit experimental data.

Multiplicity and trapped surface area

Main conjecture: multiplicity is proportional to entropy

$$S \sim N$$

Gubser et al.; 0805.1551

On experiments can be measured only N_{ch} : $N \sim N_{ch}$

B. B. Back et. al., 0210015[nucl-ex].

Accordingly experiment the charged-particles pseudorapidity density depends on colliding energy

$$dN_{ch}/d\eta \propto s_{NN}^{0.15}$$

for Pb-Pb and Au-Au çollisions

$$dN_{ch}/d\eta \propto s_{NN}^{0.11}$$ pp collision

$E=(1/2)\sqrt{s_{NN}}$ - colliding energy for nucleon

K. Aamodt et al.[ALICE Collaboration], 1011.3916 [nucl-ex].

DISCREPANCY

The simple holographic model gives

$$dN_{ch}/d\eta \propto s_{NN}^{2/3}$$

The mininal black hole entropy can be estimated by trapped surface area

$$S \geq S_{trapped} = A_{trapped} / 4G_N$$

The trapped surface is surface whose null normals all propagate inward.

S. W. Hawking and D. Page, Thermodynamics Of Black Holes In Anti-de Sitter Space, Commun. Math. Phys. 87 (1983) 577.

C. S. Pe¸ca, J. P. S. Lemos, 9805004 [gr-qc]

- $\mathcal{N}=4$ SYM is not QCD

- For holographic description of QCD a modified AdS_5 is used to study the dependence of entropy on energy

Gursoya, Kiritsis et al.,0707.1324, 0707.1349

- Early the modification of AdS_5 space-time by introduction of wrapping factor was appied for shock waves

Kiritsis, 1111.1931

- The collisions of shock waves with masses averaged over transversal surfaces are named wall-wall collisions.

- We describe heavy-ion collisions by the wall-wall shock wave collisions in AdS_5 (or in its modification)

S. Lin, E. Shuryak, 1011.1918[hep-th]

I. Y. Aref'eva, A. A. Bagrov and E. O. Pozdeeva, Holographic phase diagram of quark-gluon plasma formed in heavy-ions collisions," JHEP 1205, 117 (2012)

- In the modified 5D spaces we consider the wall-wall collisions.

Einstein equation

The Einstein equation for particle in dilaton field has the form:

$$\left(R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R\right) - \frac{g_{\mu\nu}}{2}\left(-\frac{4}{3}(\partial\Phi_s)^2 + V(\Phi_s)\right) - \frac{4}{3}\partial_\mu\Phi_s\,\partial_\nu\Phi_s - g_{\mu\nu}\frac{d(d-1)}{2L^2} = 8\pi G_5 J_{\mu\nu},$$

where $J_{++} = \dfrac{E}{b^3(z)}\delta(x^1)\delta(x^2)\delta(z-z_*)\delta(x^+).$

Shock wave metric modified by wrapping factor

$$ds^2 = b^2(z)\left(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi(z,x^1,x^2)\delta(x^+)(dx^+)^2\right)$$

Aref'eva I.Ya. 0912.5482[hep-th]

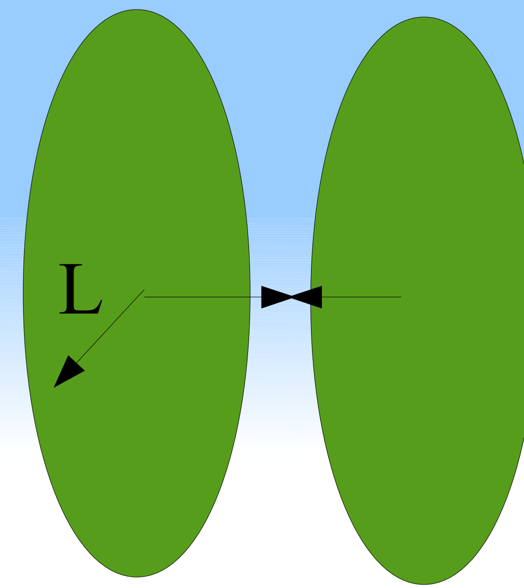
M. Hotta, M. Tanaka, Shock-wave geometry with nonvanishing cosmological constant, Class. Quantum Grav. **10,** 307, 1993

Using shock ansatz we reduce the Einstein equation to the differential equation for shock wave profile and two equations defining the connection of field and field potential with b-factor:

$$\left(\partial^2_{x^1}+\partial^2_{x^2}+\partial^2_z+\frac{3b'}{b}\partial_z\right)\phi(z,x_\perp)=-16\pi G_5\frac{E}{b^3}\,\delta(x^1)\delta(x^2)\delta(z_*-z)$$

$$V(\Phi_s)=\frac{3}{b^2}\left(\frac{b''}{b}+\frac{2(b')^2}{b^2}-\frac{4b^2}{L^2}\right) \qquad \Phi'_s=\pm\frac{3}{2}\sqrt{\left(\frac{2(b')^2}{b^2}-\frac{b''}{b}\right)}$$

Shock wave profile equation for flat objects with masses uniformly distributed collisions



- mass distributed over the domain wall

$$\left(\partial_z^2 + 3\frac{b'}{b}\partial_z \right) \phi^W(z) = -16\pi G_5 \frac{E}{b^3}\delta(z_* - z)$$

- mass distributed over the finite region with radius L

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z \right) \phi^w(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z_* - z), \qquad E^* = \frac{E}{L^2}$$

- The equations coincide up to a constant factor L^2
- The solutions to equations coincide up to constant too L^2

$$\phi^w(z) = \frac{\phi^W(z)}{L^2}$$

Trapped surface

- We apply the profile of schock wave with averanged mass for consideration of the black hole formation in AdS_5.

- We identify the black hole creation with trapped surface formation.

- In the case with averaged mass there is condition to the schock wave profile at the trapped surface (in boundary points)

$$(\partial_z \phi^w)^2 \,|_{TS} = 4$$

- The trapped surface formation condition we will apply to obtain trapped surface boundary points

The trapped surface area is calculated as follows

$$S_{trap} = \frac{1}{2G_5}\int_C \sqrt{det|g_{AdS_3}|}dz d^2x_\perp$$

where $\det|g_{\mathrm{AdS_3}}|$ is the metric determinant of AdS_3

The relative area *s* depend with the formula

$$s = \frac{S_{trap}}{\int d^2x_\perp}$$

AdS space-time modification

- For the standard AdS space-time b(z) factor has form b(z)=L/z
- We modify AdS space-time using wrapping factors types

$$b = \left(\frac{L}{z - z_0}\right)^a$$

$$b = \frac{L}{z} \exp\left(-\frac{z^2}{R^2}\right)$$

$$b = \exp\left(-\frac{z}{R}\right)$$

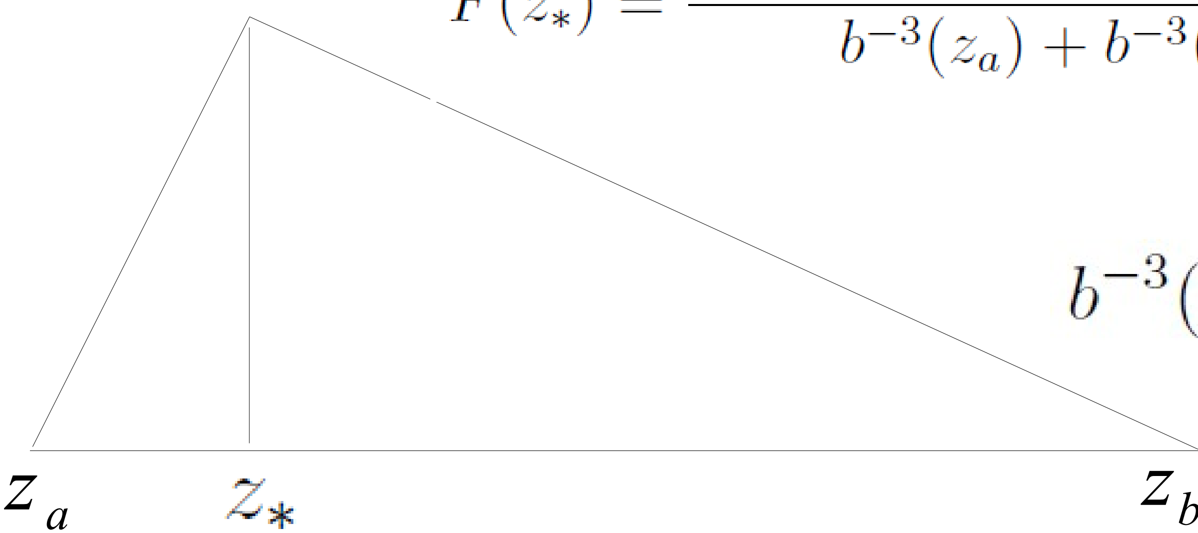
$$b = \left(\frac{L}{z}\right)^a \exp\left(-\frac{z^2}{R^2}\right)$$

where a>0, R=1 fm, $L \approx 4.4$ fm

Relations between trapped surface boundary and collision points

Using the solution to general form of domain equation (for any wrapping factor) and the trapped surface conditions we obtain the relations

$$F(z_*) = \frac{b^{-3}(z_b)F(z_a) + b^{-3}(z_a)F(z_b)}{b^{-3}(z_a) + b^{-3}(z_b)} \quad ;\partial_z F(z) = b^{-3}(z)$$



$$b^{-3}(z_a) = \frac{b^{-3}(z_b)}{\frac{8\pi G_5 E}{L^2} b^{-3}(z_b) - 1}$$

between trapped surface boundary ($z_a < z_b$) points and collision point z_*

Exponential wrapping factor

For the wrapping factor $b = \exp\left(-\frac{z}{R}\right)$ we have obtained

following relations between boundary points and collision point

$$Z_A = \frac{L^2}{16\pi G_5 E} \cdot \frac{Z_B}{Z_B - \frac{L^2}{16\pi G_5 E}}, \qquad Z_0 = \frac{L^2}{8\pi G_5 E}$$

$$Z_0 = \exp\left(\frac{3z_*}{R}\right), \quad Z_A = \exp\left(\frac{3z_a}{R}\right), \quad Z_B = \exp\left(\frac{3z_b}{R}\right)$$

For the considered case the collision point is fixed by energy.

The relative area of trapped surface dened by

$$s = \frac{3}{2RG_5}\left(\frac{1}{\exp\left(\frac{3z_a}{R}\right)} - \frac{1}{\exp\left(\frac{3z_b}{R}\right)}\right) = \frac{3}{2RG_5}\left(\frac{1}{Z_A} - \frac{1}{Z_B}\right)$$

The maximum entropy value is obtained for $Z_b \gg 1$, in this approximation

$$Z_a \sim \frac{L^2}{16\pi G_5 E}, \qquad s \sim \frac{24\pi E}{RL^2}$$

The entropy dependence on energy is linear for the exponential wrapping factor

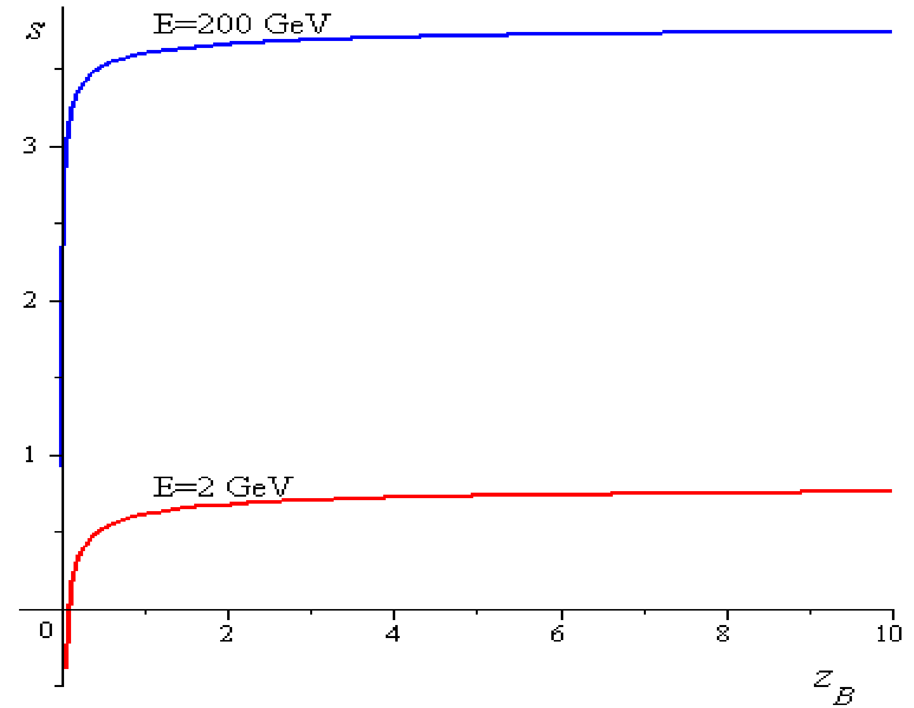
Power-law wrapping factor

Power-law b-factor $b = (L/z)^a$ gives following low boundary point and the collision point

$$z_A = \left(\frac{{z_B}^{3a}}{-1 + {z_B}^{3a} C^2}\right)^{\frac{1}{3a}}, \quad z_* = \left(\frac{{z_A}^{3a} {z_B}^{3a} (z_B + z_A)}{{z_A}^{3a} + {z_B}^{3a}}\right)^{\frac{1}{3a+1}}, \quad C^2 = \frac{8\pi G_5 E}{L^{3a+2}}$$

$$s = \frac{1}{2G_5(3a-1)}\left(z_A\left(\frac{L}{z_A}\right)^{3a} - z_B\left(\frac{L}{z_B}\right)^{3a}\right)$$

and relative area of trapped surface



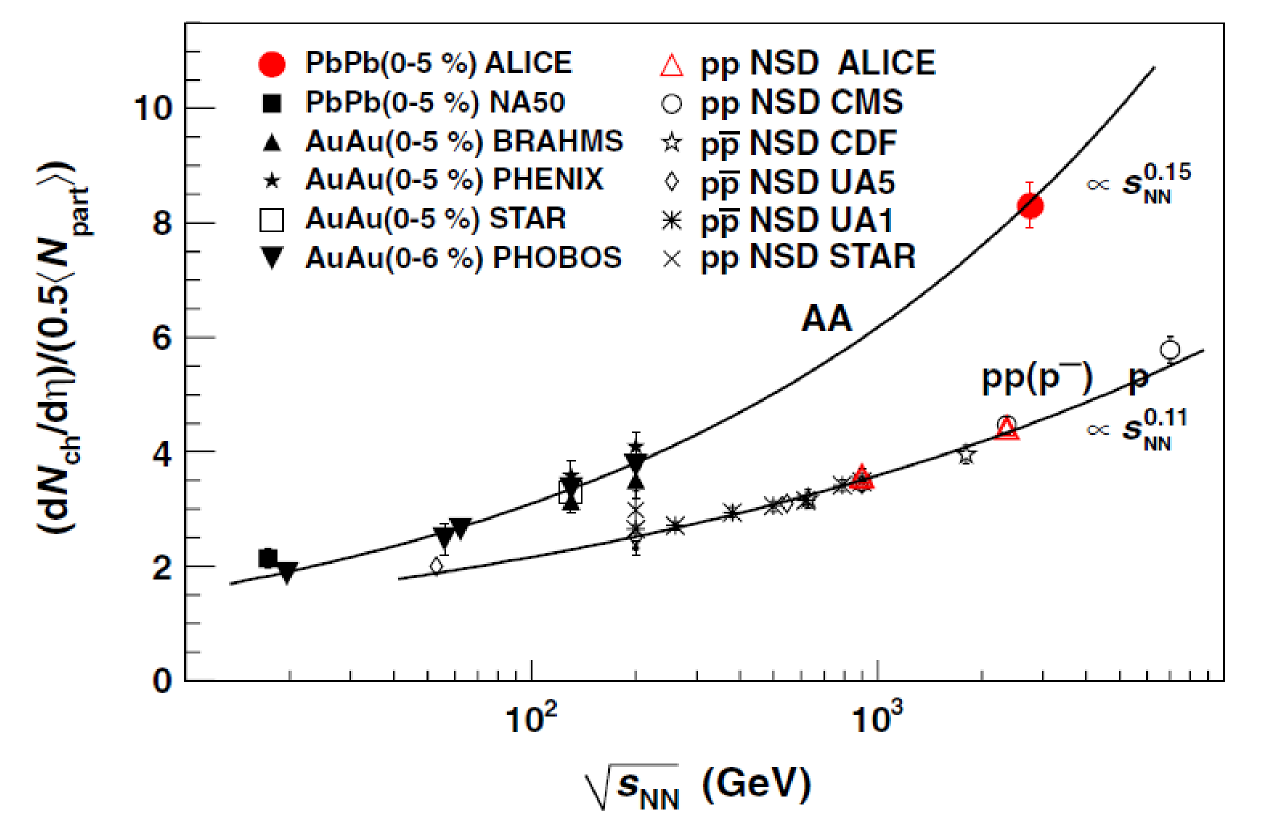
The maximal entropy value will at $Z_b \gg 1$ in assumption $3a > 1$

The maximal entropy value is relate only with energy and constant parameters

$$s\,|_{z_b\to\infty}=\frac{L^{3a}}{2G_5(3a-1)}z_A^{1-3a}=\frac{L}{2G_5}\left(\frac{8\pi G_5}{L^2}\right)^{\frac{3a-1}{3a}}E^{\frac{3a-1}{3a}}$$

For the power wrapping factor the entropy increase as $E^{1-1/3a}$

The multiplicity of particles produced in collisions of heavy ions (PbPb-and AuAu-collisions) dependents on energy as $s_{NN}^{0,15}$ ($E^{0.3}$) in the range $10-10^3$ GeV.



K. Aamodt et al. [ALICE Collaboration], arXiv:1011.3916 [nucl-ex].

The model with power-law wrapping factor can coinside with experimental data at $a \approx 0.47$

Simple mixed factor

Mixed factor of the form $b = \frac{L}{z}\exp\left(-\frac{z^2}{R^2}\right)$ gives the another relative area of trapped surface energy dependence

$$s = \frac{L^3}{2G_5}\left(-\frac{1}{2\exp\left(\frac{3z_b^2}{R^2}\right)z_b^2} + \frac{1}{2\exp\left(\frac{3z_a^2}{R^2}\right)z_a^2} + \frac{3\,\mathrm{Ei}\left(1,\frac{3z_b^2}{R^2}\right)}{2R^2} - \frac{3\,\mathrm{Ei}\left(1,\frac{3z_a^2}{R^2}\right)}{2R^2}\right)$$

which has the maximal value at $z_b \to \infty$

$$s\,|_{z_b\to\infty} = \frac{3}{4}\frac{L^3}{G_5R^2}\left(-\mathrm{Ei}\left(1,\frac{3z_a^2}{R^2}\right) + \frac{1}{3}\frac{R^2}{\exp\left(\frac{3z_a^2}{R^2}\right)z_a^2}\right)$$

and roughly is $E^{\frac{2}{3}}(1 + 0.007\ln E) - 3$ at $10\mathrm{GeV} \le E < 1\,TeV$

Complicate mixed factor

The wrapping factor $b = \left(\frac{L}{z}\right)^a \exp\left(-\frac{z^2}{R^2}\right)$

gives the most complicate relative area of trapped surface energy dependence

$$s = \frac{F(z_B) - F(z_A)}{2G_5}$$

$$F(z) = \frac{\left(\frac{L}{z}\right)^{3a} z \exp\left(-\frac{3z^2}{2R^2}\right)\left(2\left(\frac{3z^2}{R^2}\right)^{\frac{3a-1}{4}} \mathrm{M}\left(\frac{-3a+1}{4}, \frac{3(-a+1)}{4}, \frac{3z^2}{R^2}\right) + 3(1-a)\exp\left(-\frac{3z^2}{2R^2}\right)\right)}{3\,(-1+3\,a)\,(-1+a)}$$

$$\mathrm{M}(\mu,\nu,z) = \exp\left(-\frac{z}{2}\right) z^{\frac{1}{2}+\nu}\, {}_1F_1\left(\frac{1}{2}+\nu-\mu, 1+2\nu, z\right)$$

Wich has maximal value at $z_B \to \infty$: $S \to \frac{-F(z_A)}{2G_5}$

The entropy can be roughly estimate at a=1/2 such as

$$S \sim E^{0.3}(1 + C_1(\ln(E + 100))) - C_2$$

$C_1 = -0.738$, $C_2 = 0.393$ at $10 < E < 100$ GeV

$C_1 = -0.073$, $C_2 = 0.827$ at $100 < E < 1000$ GeV

Conclusions

- The black holes formation in the domain wall-wall collisions is investigated in the wrapped AdS_5 spaces.

- The several b-factor types: power-law, exponential and mixed are considered.

- The dependence of the entropy on the energy for different b-factors is analyzed.

- Our results (with the account of AdS/CFT-duality) allow to simulte the multiplicity dependence on the energy of the colliding heavy-ions in agreement with experimental data

$$b = (L/z)^a \quad , \quad a \approx 0.47 \quad , \quad S \sim E^{0.3} \quad , \quad s_{NN}^{0,15}$$

The additional logarithms appear when considering the mixed factor.

Thank you for attention!