*III International Conference MQFT devoted to 70th birthday of A.N.Vasiliev, St-Petersburg, 22.October 2010*

# Black Holes in Brane Worlds

**A.A.Andrianov,**
**St. Petersburg State University and ICCUB**

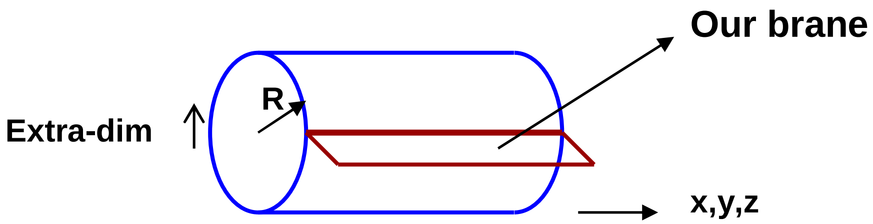
***in collaboration with*** **M.A.Kurkov**

- **What is a Brane World, what kind of Brane Worlds are suitable to generate mini black holes (BH) and what to make them of?**
- **If the suitable Worlds are higher-dimensional then what type of BH geometry one can expect to detect?**
- **Bulk BH vs Brane BH .**
- **Black stars induced by matter on a brane: exact solutions.**

**Based on** arXiv:1008.2705, PRD in print

# Extra dimensions

- Extra dimensions may be small, compact to stay invisible and with gravity background → curvature of extra-dim space to be not essential (~ Kaluza-Klein program)




- On a torus: Fourier expansion → tower of massive KK states of gravitons. Massless gravity and matter is uniformly distributed on cylinder = “bulk”,

$$m^2_{KK} = {}^{(4)}p^{\,2} \equiv \omega^2 - {}^{(3)}\mathbf{p}^{\,2} = \frac{\mathbf{n}^2}{R^2}\,.$$

- Alternatively, gravity on the bulk but matter on a brane = a hypersuface at a point in extra dimensions filled by matter, cosmological constant = brane tension and gravity induced from the brane

# Size of extra dimensions

**Planck scale transmutation due to large volume of extra dimensions**

$$M_{Pl}^2 = 8\pi \bar{M}_{Pl}^2 \qquad \bar{M}_{Pl}^2 = M_*^{n+2}(2\pi R)^n \qquad M_{Pl} \simeq 10^{16} TeV \Rightarrow M_* \simeq 1 TeV$$

**Gravity - <u>in 4 dim</u> and <u>reduced from (4+n) dim</u>**

**Table-top experiments!**

$\boxed{\text{Modified Newton law}}$

**<u>n = 1</u>**

$$V(\vec{x}) = \begin{cases} -\frac{4}{3}\frac{G_N M}{r}, & r \gg R; \\ -\frac{M}{6\pi^2 M_*^3 r^2}, & r \ll R. \end{cases}$$

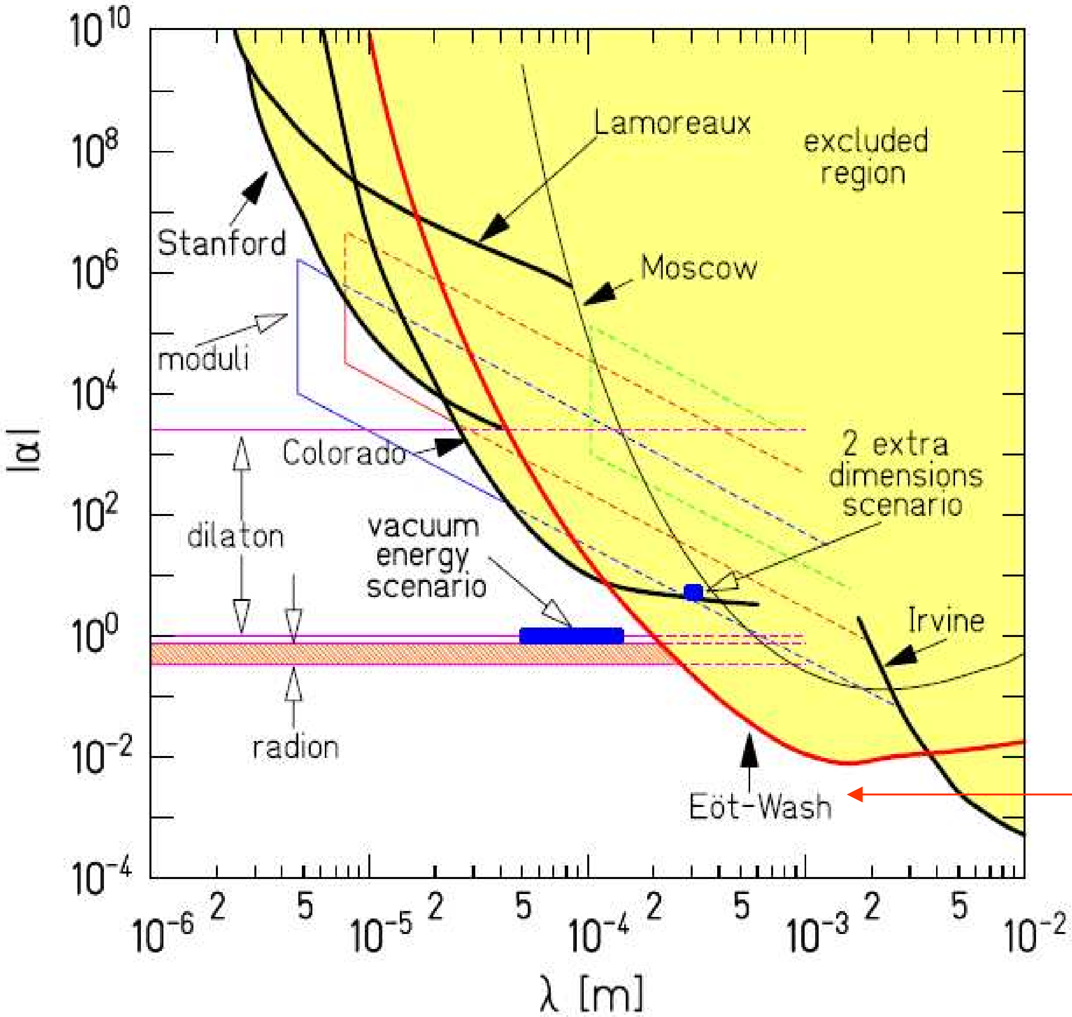
$\underline{n > 1}$

$$V(r) \sim \int d^{3+n}q \frac{M}{M^{2+n}q^2} \sim -\frac{M}{M^{2+n}r^{n+1}} \qquad r \ll R$$

**Black Hole metric?** **U(r) = 1+ 2 V(r) =** $g_{00}$

- Spectrum of KK gravitons $\longrightarrow$ modification of the Newton law at short distances

$$V(r) = -\frac{G_N m}{r}\left(1 + \alpha\, e^{-\frac{r}{\lambda}}\right)$$




$\lambda < 50\mu m\,(95\%CL)$

Regions in the $\alpha - \lambda$ plane excluded by table top searches for deviations from Newtonian gravity

## Search for Large Extra Dimensions via Single Photon plus Missing Energy Final States at sqrt(s) = 1.96 TeV

**KK scenario**

$$\bar{M}_{Pl}^2 = M_*^{n+2}(2\pi R)^n$$

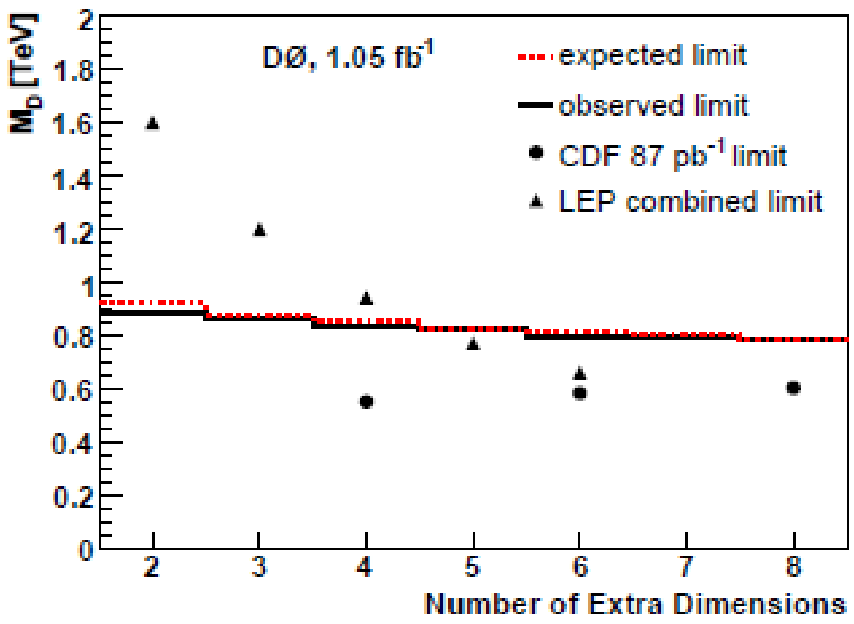
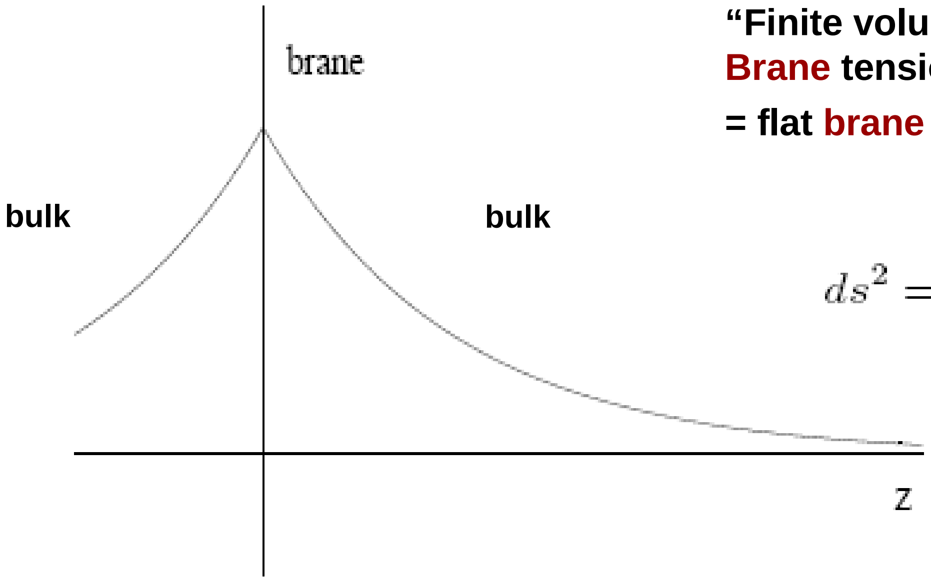



FIG. 3: Expected and observed lower limits on $M_D$ for LED in the $\gamma + \not{E}_T$ final state. CDF limits with 87 $\mathrm{pb}^{-1}$ of data [3], and the LEP combined limits [6] are also shown.

D0 Collaboration Phys.Rev.Lett.101:011601,2008

## Matter on the brane + gravity in the bulk: large extra dim

L.Randall and R.Sundrum (II)




"Finite volume of extra dimensions" $\sim 1/k$

**Brane** tension vs **Bulk** cosmological constant

= flat **brane** but AdS **bulk**

$$ds^2 = e^{-2k|y|}[\eta_{\mu\nu} + h_{\mu\nu}(x)]dx^\mu dx^\nu - dy^2$$

**Graviton is localized around the brane!**

$$\boxed{\text{Modified Newton law}}$$

$$V(\vec{x}) = \begin{cases} -\frac{G_N M}{r}\left(1 + \frac{2}{3k^2 r^2} + \mathcal{O}(1/r^3)\right), & r \gg 1/k; \\ -\frac{M}{6\pi^2 M_*^3 r^2}, & r \ll 1/k. \end{cases}$$

Exp.

**k > 0.004 eV**

**Black Hole metric? U(r) = 1+ 2 V(r) =** $g_{00}$

## Synthesis of brane KK and RS

**Vacuum of low energy supergravity** $AdS_5 \times S^{(5)}$ ← **Sphere**

$$S^{(5)} \to T^{(5)} = \left[ \times S^{(1)} \right]^5$$ **or torus**

$$M_*^8 \cdot L^5 \frac{1}{k} = M_{Pl}^2;$$ → $$L = \sqrt[5]{\frac{M_{Pl}^2 \cdot k}{M_*^8}} < 1\mathrm{fm}$$ **Nuclear size!**

**if** $M_* \sim 1TeV$ k > 0.004 eV $M_{Pl} = 10^{16}TeV$

**<u>A large dimension</u>** **L << 1/k**

## Static Black Holes in our 3+1 world

$$ds^2 = U(r)dt^2 - \frac{1}{U(r)}dr^2 - r^2 d\Omega_2^2$$ **Area gauge**

**Black Hole when its mass inside its horizon (no-return surface)** $U(r_h) = 0$

**Schwarzschild (1916)**

$$U_{Sch}(r) = 1 - \frac{2G_N M}{r} \qquad 2G_N M = \frac{2M}{M_{Pl}^2} \equiv r_g$$

**It gives the exact Newton law for gravitational potential** $g_{00}$

## Reissner-N”ordstrom (1916)

$$U_{R-N}(r) = 1 - \frac{2G_N M}{r} + \frac{G_N Q^2}{r^2} \qquad r_g = G_N M\left(1 \pm \sqrt{1 - \frac{Q^2 M_{Pl}^2}{M^2}}\right)$$

**Tidal charge Q^2 < 0? The signature of fifth dim.**

**For particles/partons in collider experiments**

$$M \sim 1GeV \qquad Q^2 \sim e^2 \sim 0.1 \qquad \frac{M^2}{M_{Pl}^2} \sim 10^{-38} \lll Q^2 \Rightarrow \boxed{U_{R-N}(r) \neq 0}$$

**No horizon in 3+1 ! No BH description for SM hadrons**
**But a large tidal charge may enlarge the horizon radius**

# Higher-dimensional Black Holes

**Static BH**

$$ds^2 = U(r)dt^2 - \frac{1}{U(r)}dr^2 - r^2 d\Omega^2_{n-1}$$

**Schwarzschild-Tangherlini**

$$\boxed{U_{Sch-Tan}(r) = 1 - \frac{r_g^{n+1}}{r^{n+1}}}$$

$$r_g = \frac{1}{\sqrt{\pi}M_*}\left(\frac{8\Gamma(\frac{n+3}{2})}{(n+2)}\frac{M}{M_*}\right)^{1/(n+1)}\Bigg|_{n=(1,...7)} \le \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{1/(n+1)}$$

**Evidently for any particles with masses** $M < M_* \sim 1TeV$

**their horizon is less than the Plank scale** $r_g < 1/M_*$

**Therefore SM particles by no means can be considered as classical BH, their apparent horizons can be only treated in Quantum Gravity approach**

**Only the objects with** $M \gg M_* \sim 1TeV$ $\quad r_g \gg 1/M_*$

**can be considered as true classical black holes:** **M > 3-5 TeV?**

# From bulk to brane

If the matter is located on a brane then the Sch.-Tan. metric is generated by a point-like particle at r = 0 whereas we want to deal with matter distribution on the brane to proceed to 3+1 dim world

Let's introduce cylindrical coordinates $r, \theta \to \rho = r\cos\theta,\ z = r\sin\theta$

Does it exist a matter distribution which provides the interpolation of a section of S-T geometry at z = 0 at intermediate distances to asymptotical Reisner-N"ordstrom geometry?

$$U_{brane}(\rho, z = 0)\Big|_{\rho \gg r_g} \overset{?}{\simeq} 1 - \frac{2G_N M}{\rho} + \frac{G_N Q^2}{\rho^2} - \frac{r_g^{n+1}}{\rho^{n+1}}$$

This conjecture has not yet been verified and proved !

Anyways the last "tidal" term will certainly dominate over R-N one

even for n= 1 and for $M \gg M_* \sim 1TeV$ it could trigger black hole formation

## Bulk BH vs. Brane BH

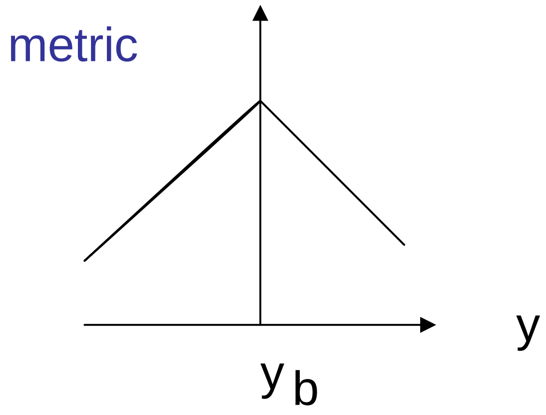
### Einstein 5

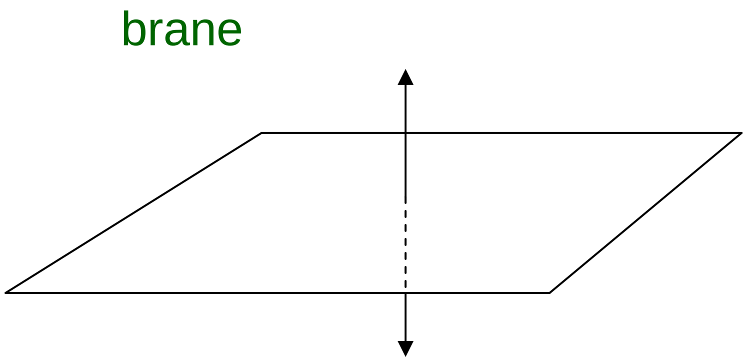
$$^{(5)}G_{AB} = \frac{^{(5)}T_{AB}}{\kappa_5}; \;\Rightarrow\; ^{(5)}R = -\frac{2}{3}\frac{^{(5)}T}{\kappa_5}; \quad \kappa_5 \equiv M_*^3,$$

**Energy-momentum tensor**

### Einstein 5 → SMS 4

$$^{(5)}G_{\mu\nu} = G_{\mu\nu} - \partial_y K_{\mu\nu} + g_{\mu\nu}\partial_y K + K K_{\mu\nu} - 2K_\mu^\sigma K_{\sigma\nu} - \frac{1}{2}g_{\mu\nu}(K^2 + K_{\sigma\rho}K^{\rho\sigma});$$




**Gaussian normal coordinates**

**Orbifold** ⟷ **R-S II**

$$g_{\mu\nu}(y_b + y) = g_{\mu\nu}(y_b - y);$$

**Israel-… matching conditions**

$$K^{+}_{\mu\nu} = -K^{-}_{\mu\nu} = -\frac{1}{2\kappa_5}\left(S_{\mu\nu} - \frac{1}{3} g_{\mu\nu} S\right).$$

**Conformal Weyl tensor**

$$R_{ABCD} = \frac{1}{N+2}(g_{AC}R_{BD} - g_{AD}R_{CB} + g_{BD}R_{AC} - g_{BC}R_{AD}) -$$

$$-\frac{1}{(N+2)(N+3)}(g_{AC}g_{BD} - g_{AD}g_{BC})R + C_{ABCD};$$

**traceless**

# SMS (Shiromizu-Maeda-Sasaki) equation

$$\boxed{G_{\mu\nu} =} \frac{2}{3\kappa_5}\Big( {}^{(5)}T_{\mu\nu} - g_{\mu\nu}\Big( {}^{(5)}T_{55} + \frac{1}{4}\,{}^{(5)}T\Big)\Big) - KK_{\mu\nu} + K^{\sigma}_{\mu}K_{\sigma\nu} + \frac{1}{2}g_{\mu\nu}(K^2 - K^{\sigma\rho}K_{\sigma\rho}) - E_{\mu\nu}$$

$$\boxed{= \frac{\Lambda}{\kappa}g_{\mu\nu} + \frac{1}{\kappa}\tau_{\mu\nu} + \frac{1}{\kappa\lambda}\Sigma_{\mu\nu} - E_{\mu\nu},}$$

**Gravity from the bulk**

$$\Sigma_{\mu\nu} = \frac{1}{4}\Big( - 2\tau\tau_{\mu\nu} + 6\tau^{\sigma}_{\mu}\tau_{\sigma\nu} + g_{\mu\nu}(-3\tau^{\sigma\rho}\tau_{\sigma\rho} + \tau^2)\Big).$$

$$E_{\mu\nu} \equiv {}^{(5)}C^{5}_{\mu 5\nu}$$

$$\boxed{\Lambda = \frac{1}{2}(\Lambda_5 + \frac{\lambda^2}{6\kappa_5}); \quad \kappa \equiv \frac{6\kappa_5^2}{\lambda},}$$

**Brane is flat**

$$\Lambda_5 = -\frac{\lambda^2}{6\kappa_5}; \ \lambda = \pm\sqrt{6\kappa_5\Lambda_5}; \ \kappa_5 \equiv M_*^3.$$

# Solution ??

static vacuum,

$$\mathcal{E}_{\mu\nu} = \left(\frac{q}{\widetilde{M}_{\rm P}^2}\right)\frac{1}{r^4}\left[u_\mu u_\nu - 2r_\mu r_\nu + h_{\mu\nu}\right] .$$

**Weyl component, controls communication to extra dimension !**

$$g_{tt} = (g_{rr})^{-1} = 1 - \left(\frac{2M}{M_{\rm P}^2}\right)\frac{1}{r} + \left(\frac{q}{\widetilde{M}_{\rm P}^2}\right)\frac{1}{r^2} ,$$

$q = Q\widetilde{M}_{\rm P}^2$ *is a dimensionless tidal charge* $< 0$

**Where from is Q? From 5th dimension!**

**But this solution is accompanied by penetrating the matter to the bulk!**

**A lack of exact solutions with matter localized on the brane!?**

- Thus BH creation may be a consequence of strong at short distances attainable in high energy experiments if our space is a three-brane in an extra-dimensional space.

- Exact description of BH geometry is needed when matter is located on a brane but gravity propagates into extra space dimensions.

- What kind of solutions with horizons we seek for? Black stars! Indeed the matter (quarks and gluons in LHC experiments) must be smoothly distributed without any strong singularities hidden under horizon.

- Therefore one expects that in high energy collisions rather black stars are formed with matter both inside and outside an event horizon stuck to our three-dim space.

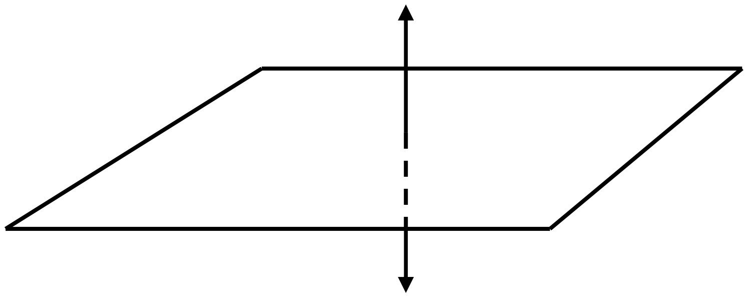
- Stress-energy tensor structure.
  - The Einstein equations in the bulk read,

$$^{(5)}G_{AB} = \kappa_5 T_{AB}, \quad T_{AB} = \delta^{\mu}_{A}\delta^{\nu}_{B}\tau_{\mu\nu}\delta(z)$$

with $\kappa_5 = 1/M_*^3$ and $M_*$ is a Planck scale in five dimensions.

  - In order to define $\tau_{\mu\nu}$ let us introduce extrinsic curvature tensor $K_{\mu\nu}$.

$K_{\mu\nu} = -\frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial z}$ *valid in the Gaussian normal coordinates*$(i.e. g_{zz} = -1, g_{\mu z} = 0)$ *only*!



**Orbifold** $K^{+0}_{\mu\nu} = -K_{\mu\nu -0}$

- $\tau_{\mu\nu}$ is defined by the Israel-Lanczos junction conditions,

$$[g_{\mu\nu}K - K_{\mu\nu}]^{+0}_{-0} = \kappa_5 \tau_{\mu\nu}.$$

- $K^{+0}_{\mu\nu}$, and $K_{\mu\nu -0}$ are the extrinsic curvature tensors of hypersurfaces $z = +0$ and $z = -0$ correspondingly.

- General construction.
  - To build a brane we search for a metric $g_{AB}(x, y)$ which is a bulk vacuum solution of the Einstein equations with event horizon.
  - Suppose that:
    - a) the induced metric $g_{\mu\nu}(x, y)$ is asymptotically flat for any hypersurface $y = const$ and inherits the horizon;
    - b) in the chosen coordinate systems $g_{5B}(x, y) = 0$ and the remaining metric components provide orbifold geometry $g_{AB}(x, y) = g_{AB}(x, -y)$;
    - c) Coordinate $y$ is spacelike i.e. $g_{yy} \equiv g_{55} < 0$.
  - In order to generate a brane filled by matter we proceed the following transformation,

$$g_{AB}(x, y) \Longrightarrow g_{AB}(x, |z| + a).$$

  - Brane: $z = 0$.

# Construction of the solution.

- Preparing of the suitable coordinate system.
  - We start from the metric describing a five-dimensional static neutral black hole in Schwarzschild coordinates $\{t, r, \theta_1, \theta_2, \theta\}$,

$$g_{AB} = \mathrm{diag}\left[U(r), -\frac{1}{U(r)}, -r^2\cos^2\theta, -r^2\cos^2\theta\cos^2\theta_1, -r^2\right],$$

    where $U(r) = 1 - \frac{M}{r^2}$, $M$ is related to the Schwarzschild-Tangherlini radius $M \equiv r^2_{Sch-T}$.
  - Let's define the Gaussian normal coordinates in respect to hypersurface with space-like normal vector $\theta = 0$.
  - The vector orthonormal to this hypersurface $n^A = [0, 0, 0, 0, 1/r]$ .
  - The required change of coordinates acts on two variables $r = r(\rho, y)$, $\theta = \theta(\rho, y)$.

- Our coordinate transformation has the following form.
  - $$|y| = \int_\rho^r \frac{\mathrm{sign}((r-\rho))\, x^2}{\sqrt{(x^2-M)\,(x^2-\rho^2)}}dx, \quad \theta = \int_\rho^r \frac{\mathrm{sign}((r-\rho)y)}{\sqrt{(x^2-M)\,(x^2-\rho^2)}}dx.$$
  - We have: inside the horizon $r < \rho < \sqrt{M}$ and outside the horizon $\sqrt{M} < \rho < r$ .
  - The metric in new coordinates $\{t, \rho, \theta_1, \theta_2, y\}$, reads,

    $$g_{AB}(x,y) = \mathrm{diag}\left[U(r), -\frac{r^2 {r_\rho}^2}{\rho^2 U(r)}, -r^2\cos^2\theta, -r^2\cos^2\theta\cos^2\theta_1, -1\right],$$

    where $r = r(\rho, y)$, $\theta = \theta(\rho, y)$.
  - The final answer for black star metric and $\tau_{\mu\nu}$ has the following form:

$$g_{AB}^{final}(x,z) = g_{AB}(x,y)|_{y=|z|+a},\ \kappa_5\tau_{\mu\nu}(x,a) = \left(\frac{\partial g_{\mu\nu}}{\partial y} - g_{\mu\nu}\frac{g^{\lambda\delta}\partial g_{\lambda\delta}}{\partial y}\right)\Big|_{y=a}$$

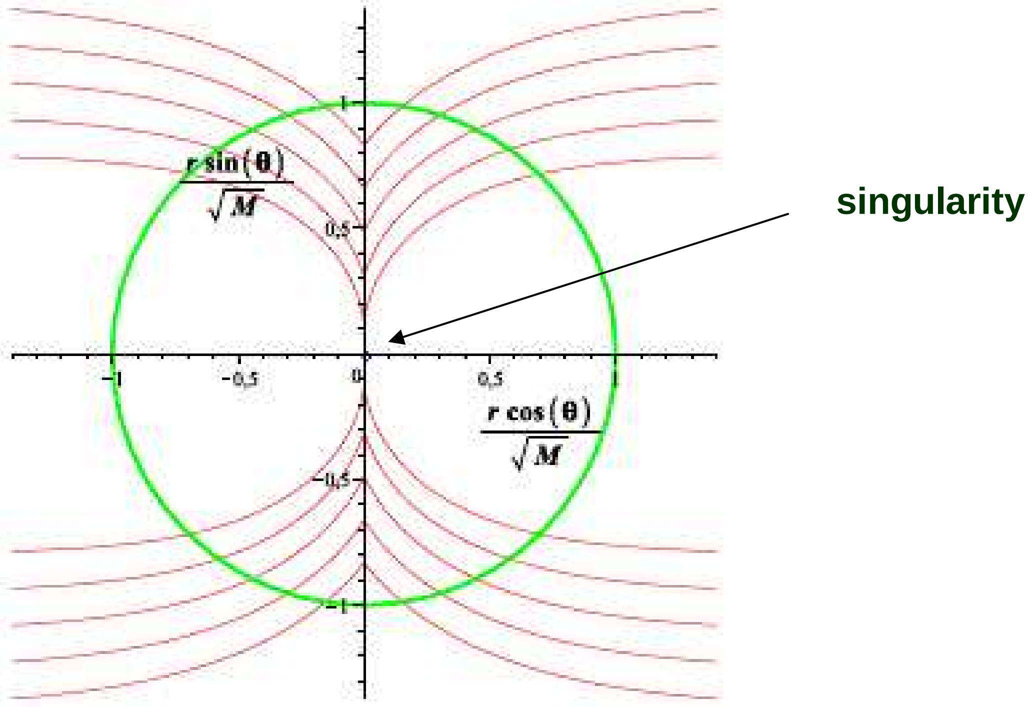



Figure 1: Pairs of hypersurfaces symmetric in respect to the horizontal axis to be glued into a brane are shown by red curves. The circle of horizon in dim $= 5$ is depicted by green line. $g_{AB}(x,y) \Longrightarrow g_{AB}(x,|z|+a)$ , $a = 0.69868\sqrt{M} \div \pi\sqrt{M}/2$

**Critical a !**

# Matter distribution.

- Effective 4-D stress-energy tensor $S_{\mu\nu}$.
  - projection of Einstein equations onto the brane: SMS equations

$$^{(4)}G_{\mu\nu} \equiv G_{\mu\nu} = \kappa_5^2 \Sigma_{\mu\nu} - E_{\mu\nu} \equiv \kappa_4 S_{\mu\nu}, \quad \kappa_4 \equiv \frac{1}{M_{Pl}^2},$$

  - where

$$\Sigma_{\mu\nu} = \frac{1}{24}\Big(-2\tau\tau_{\mu\nu} + 6\tau_\mu^\sigma \tau_{\sigma\nu} + g_{\mu\nu}\big(-3\tau^{\sigma\rho}\tau_{\sigma\rho} + \tau^2\big)\Big),$$

  - and

$$E_{\mu\nu} =^{(5)} C^A_{BCD} n_A n^C q^B_\mu q^D_\nu .$$

  - Compare with 5-D Einstein equations:

$$^{(5)}G_{AB} = \kappa_5 \delta^\mu_A \delta^\nu_B \tau_{\mu\nu} \delta(z).$$

$R(\rho, a) \equiv r(\rho, a) \cos\theta(\rho, a)$ on the brane.

- The total mass in 4+1 dimension is given by

$$\mathcal{M} = \frac{3}{16\pi\kappa_5} \int_{t=const} d^{(4)}V\, {}^{(5)}R_{AB}\xi^A m^B \equiv \int_0^\infty dR\, f_5(R).$$

- $\mathcal{M}$ does not depends on the value of parameter $a$!
- The exact calculations show that the 3-dim Komar integral, ${}^{(4)}\mathcal{M}_{eff} = 0$.

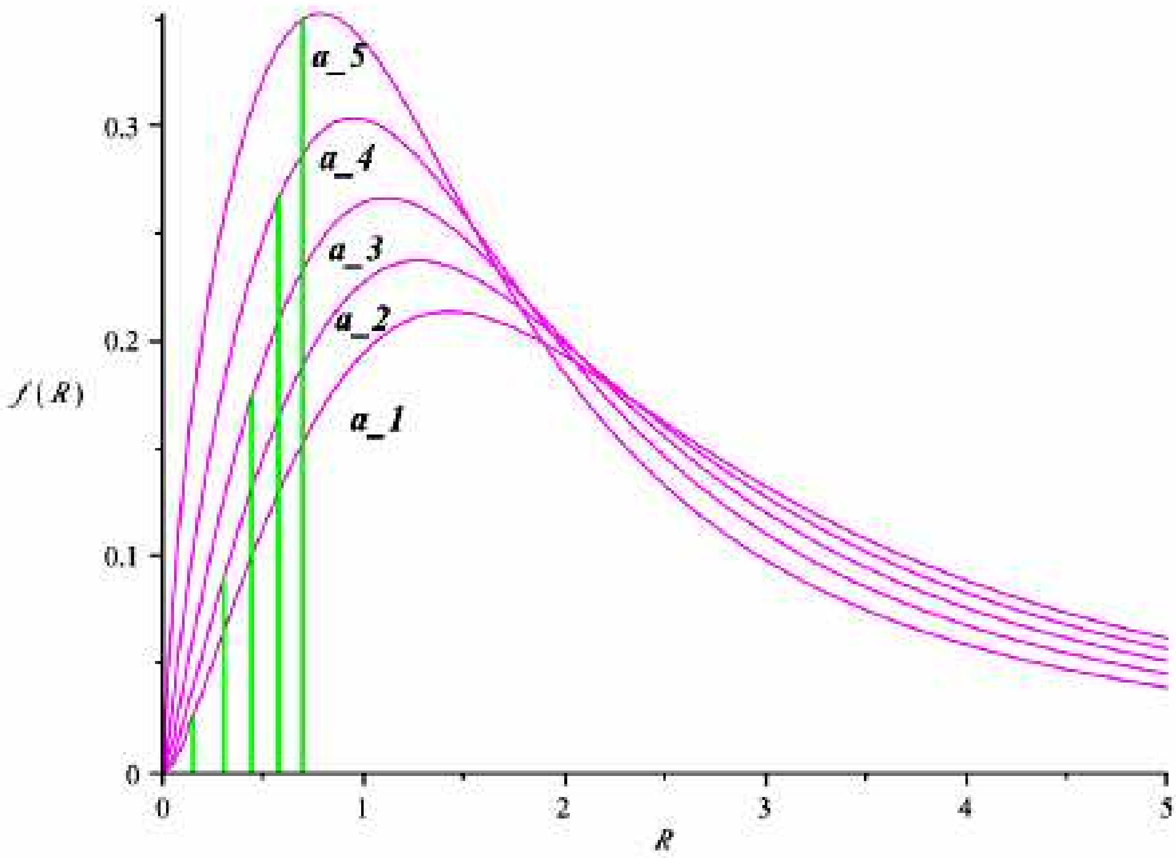



Figure 2: The matter-density radial distributions $f_5(R, a)$ on the brane with $M = 1$ are presented by a magenta colored line. The corresponding horizons are indicated by green lines.

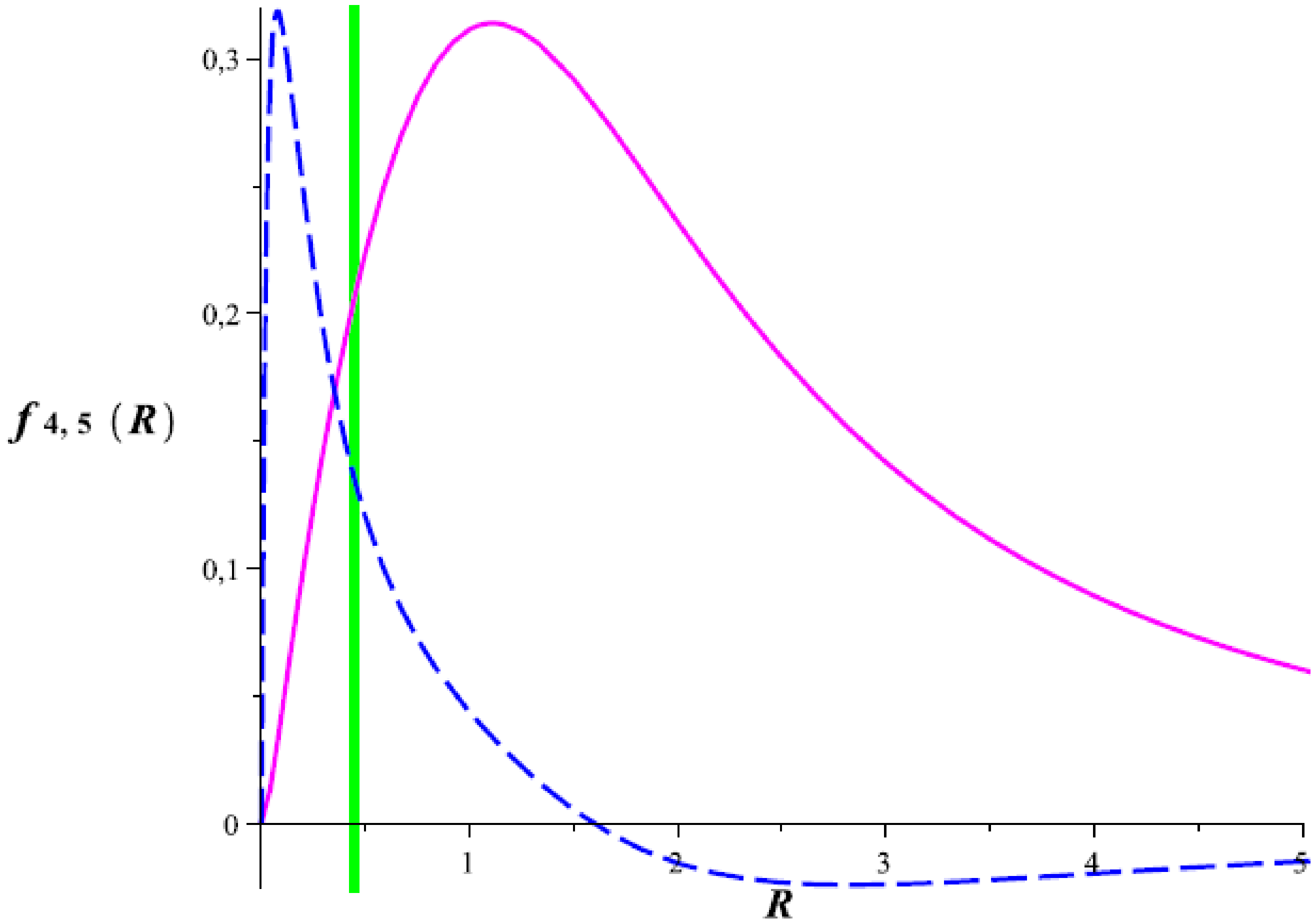



Figure 3: The matter-density radial distribution $f_5(R)$ on the brane with $a = 1.1, M = 1$ is presented for $\kappa_5 = 1$ by a magenta colored line. The effective matter-density $f_4(R)$ is shown by blue line for the value $\kappa_4 = 50$ to compare with $f_5(R)$. The horizon is indicated by green line.

# Conclusions

- We have shown that by cut-and-paste method in special Gaussian normal coordinates one can build the exact geometry of multidimensional black star with *horizon*, generated by a *smooth* matter distribution in our universe.
- In our approach, for a given total mass, the profiles of available configurations for matter distribution are governed by the parameter $a$ which is presumably related to the collision kinematics when a black object ("black hole") is created by partons on colliders.

## Black hole generation: numerical simulation

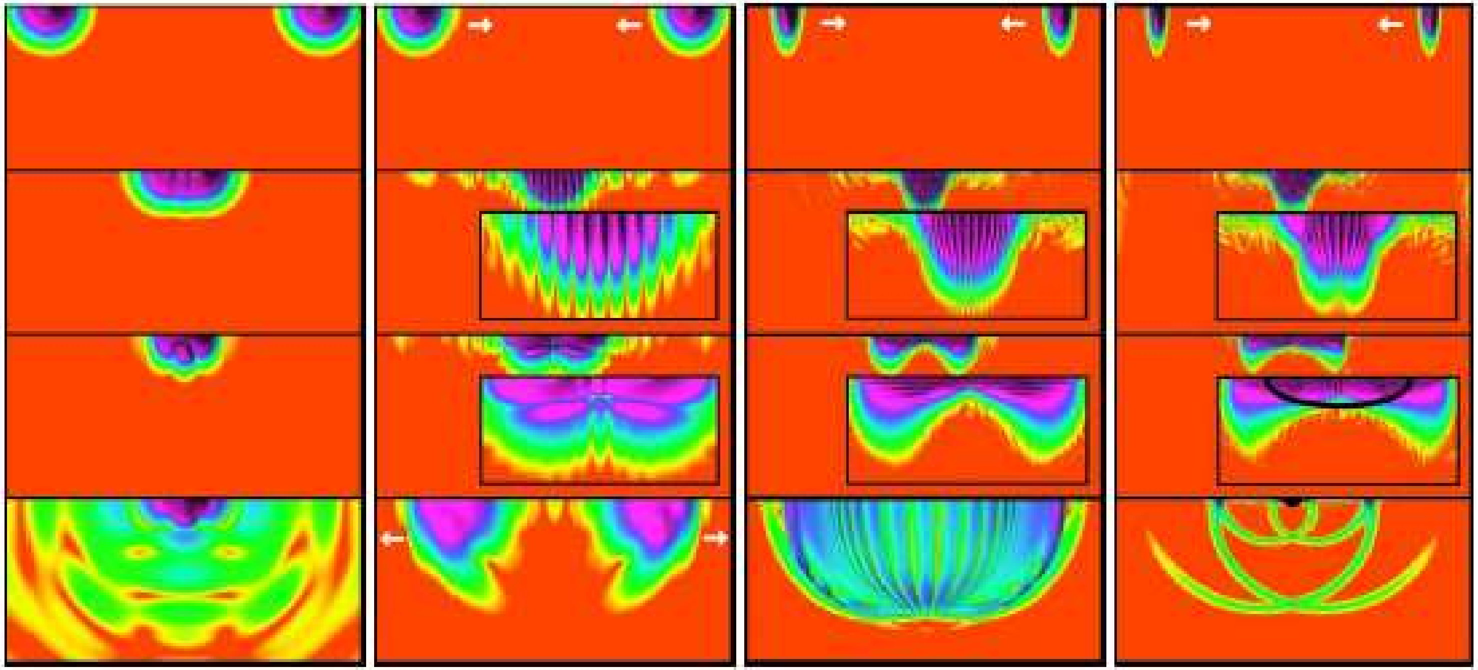


FIG. 1: The magnitude $|\phi|$ of the scalar field for collisions with $\gamma = 1, 1.15, 2.75$ and 4 (left to right). The axis of symmetry is coincident with the top edge of each panel. Four times are shown in each sequence ($t/M_0 = [0, 4390, 4500, 7480]$ for $\gamma = 1$, $[0, 254, 297, 469]$ for $\gamma = 1.15$, $[0, 141, 172, 313]$ for $\gamma = 2.75$ and $[0, 137, 168, 309]$ for $\gamma = 4$): the top panel is $t = 0$, the second panel down is when the boson stars first completely overlap, the third panel is shortly afterwards when $|\phi|$ reaches a first local maximum due to gravitational focusing, and the fourth panel is at a late time after the collision. The insets, where present, are zoom-ins of the central interaction regions. For the $\gamma = 4$ case, a black hole forms near the time of panel 3—the black line in the corresponding inset shows the shape of the apparent horizon then. In the fourth panel of this figure, the black semi-circle is the excised region inside the black hole; the small size of this compared to the apparent horizon in the previous figure is a coordinate effect—the proper area of the horizon grows with time.

Matthew W. Choptuik[1] and Frans Pretorius[2] **arXiv:0908.1780**