Field theoretic approach in kinetic reaction: Role of random sources and sinks

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III International Conference Models in Quantum Field Theory
dedicated to Alexander Nikolaevich Vassiliev's 70-th Anniversary

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Formulation of the problem

- reaction-diffusion model - single species of particles A do random walk in space and, whenever they meet, undergo the reaction $A + A \to \emptyset$ (inert) at rate λ

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- if $\tau_{dif} >> \tau_{react}$ - diffusion limited case $\Rightarrow$ fluctuation of density must be taken into account
  attempt to do it by adding a noise term as in Langevin approach:

$$\frac{\partial n}{\partial t} = D\nabla^2 n - 2\lambda n^2 + \zeta$$

  this simple-minded approach is dubious

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- Master equation: time evolution of the probability $P(\alpha)$ of finding a system in a microstate α

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- ”Second quantization” formalism: allows to rewrite the problem in the language of creation-annihilation operators in Fock space, to use QFT and finally formulate it as a functional integral with effective dynamical action
- Dynamic action of the diffusion-limited annihilation reaction $A + A \to \emptyset$

$$\begin{aligned} S_1 &= -\int_0^{\infty} dt \int d\mathbf{x} \left\{ \psi^{+}\partial_t\psi - D_0\psi^{+}\nabla^2\psi \right. \\ &+ \left. \lambda_0 D_0 \left[2\psi^{+} + (\psi^{+})^2\right]\psi^2 \right\} + n_0 \int d\mathbf{x}\, \psi^{+}(\mathbf{x}, 0) \end{aligned}$$

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- Our primary aim: to investigate influence of random drift field (turbulent fluctuations, thermal fluctuations) on dynamics of annihilation reaction, to find universal critical regimes by RG approach and decay law for average density
- we have carried out the analysis in two-loop approximation (calculated all RG functions, fixed points and its stability) for cases when statistics of velocity field is prescribed –like in rapid change Kraichnan model – or it is determined by stochastic Navier-Stokes equation. Details of our analysis and calculation will be presented on Thursday by Tomas Lucivjansky

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- no unique way to introduce random sources in the master equation corresponding to the random noise of the mean-field (Langevin) description.

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- Introduction of random sink and source: Langevin approach is not suitable for such processes $\Rightarrow$ attempt to introduce them directly to the master equation
- no unique way to introduce random sources in the master equation corresponding to the random noise of the mean-field (Langevin) description.
- the simplest choice: to model the sink by reaction $A \to X$ and source by reaction $Y \to A$, where X and Y stand for particle baths of the sink and the source

  N. G. van Kampen, Stochastic processes in physics and chemistry (North-Holland, Amsterdam, 1984)

Random sources and sinks in the master equation

- Master equation with inclusion of random sourses and sinks (homogeneous case)

$$\begin{aligned}\frac{\mathrm{d}P(t,n)}{\mathrm{d}t} &= \mu_+ V\left[P(t,n-1)-P(t,n)\right] \\ &+ \mu_-\left[(n+1)P(t,n+1)-nP(t,n)\right]\ldots\end{aligned}$$

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- μ_+ and μ_- - reaction constants of the creation and annihilation reactions
- annihilation process: transition rate - proportional to the particle number n
- creation process: transition rate - proportional to the volume of homogeneous system V
- reaction-rate equation

$$\frac{\mathrm{d}\langle n\rangle}{\mathrm{d}t} = \mu_+ V - \mu_-\langle n\rangle + \ldots$$

  $\langle n\rangle$ - mean particle number

Random sources and sinks: Doi approach

- Doi approach
  M. Doi, J. Phys. A: Math. Gen. **9** *(1976) 1465, 1479*
  see also *J.Cardy, Field Theory and Nonequilibrium Statistical Mechanics, Troisieme cycle de la Physique en Suisse Romande, Anné académique 1998-1999, semestre d'été*

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- Basis vectors of the Fock space: Usual annihilation and creation operators $\hat{a}$, $\hat{a}^+$ and the vacuum vector $|\,0\,\rangle$:

$$\begin{aligned} \hat{a}|\,0\,\rangle &= 0 \quad , \quad \hat{a}^+|\,n\,\rangle = |\,n+1\,\rangle \\ [\,\hat{a}, \hat{a}^+] &= \hat{a}\hat{a}^+ - \hat{a}^+\hat{a} = I \end{aligned}$$

normalization: $\langle\, n\,|\, m\,\rangle = n!\delta_{nm}$

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- State vector: contains all information about the system:

$$|\,\Phi\,\rangle = \sum_{n=0}^{\infty} P(t,n)|\,n\,\rangle$$

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- Master equations for a birth-death process $\Rightarrow$
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  Single evolution equation for the state vector:
- $$\frac{\mathrm{d}|\,\Phi\,\rangle}{\mathrm{d}t} = \hat{L}(\hat{a}^{+},\,\hat{a})|\,\Phi\,\rangle$$
- Liouville operator
  $$\hat{L}_g(\hat{a}^{+},\hat{a}) = \mu_{+}V\left(\hat{a}^{+} - I\right) + \mu_{-}\left(I - \hat{a}^{+}\right)\hat{a}\,,$$
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- Expectation value of any function $F(n)$ of random particle number:
  $$\langle F(t)\rangle = \sum_{n=0}^{\infty} F(n)P(t,n)$$
  Form of the functional integral over the functions of time $\tilde{a}$ and a:
  $$\langle F(t)\rangle = \int \mathcal{D}\tilde{a}\mathcal{D}a\, F_N[(\tilde{a}(t)+1)a(t)]e^{S_1}\,,$$
  $F_N(\tilde{a}a)$ - normal form of the operator $F(\hat{a}^{+}\hat{a})$

Random sources and sinks: Doi approach

- S_1 - dynamic action

$$S_1(\tilde{a}, a) = \int_0^\infty dt\, \left[-\tilde{a}(t)\partial_t a(t) + \mu_+ V \tilde{a}(t) - \mu_- \tilde{a}(t) a(t)\right] \ldots$$

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- Assumption: transition rates $\mu_\pm$ - random functions uncorrelated in time
  Their probability distribution: moments $\langle \mu_\pm^n \rangle = E_{\pm\,,n}$

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  volume V: volume element attached to the lattice site
  Transition rates at each time instant and lattice site $\mu_{\pm\,\alpha,i}$ - independent random variables

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- Average of the expectation value over the distribution of random sources:

$$\prod_{\alpha,i} \langle e^{\mu_{+\,,\alpha,i} V \tilde{a}_{\alpha,i} \Delta t - \mu_{-\,,\alpha,i} \tilde{a}_{\alpha,i} a_{\alpha,i} \Delta t} \rangle$$

- Cumulant expansion:

$$
\begin{aligned}
\langle e^{\mu b \Delta t} \rangle &= 1 + b\Delta t E_1 + \frac{1}{2} E_2 (b\Delta t)^2 + \frac{1}{6} E_3 (b\Delta t)^3 + \cdots \\
&= e^{b\Delta t E_1 + \frac{1}{2}\left(E_2 - E_1^2\right)(b\Delta t)^2 + \frac{1}{6}\left(E_3 - 3E_1 E_2 + E_1^3\right)(b\Delta t)^3 + \cdots}
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\begin{aligned}
\prod_{\alpha,i} \langle e^{\mu_{+,\alpha,i} V\tilde{a}_{\alpha,i}\Delta t} \rangle &= e^{\sum_\alpha \sum_i \left[\Delta t E_{+1} V\tilde{a}_{\alpha,i} + \frac{1}{2}\left(E_{+2} - E_{+1}^2\right)(V\tilde{a}_{\alpha,i}\Delta t)^2\right]} \\
& \quad e^{\sum_\alpha \sum_i \left[\frac{1}{6}\left(E_{+3} - 3E_{+1}E_{+2} + E_{+1}^3\right)(V\tilde{a}_{\alpha,i}\Delta t)^3 + \cdots\right]}
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Random sources and sinks: Doi approach

- Continuum limit: function $\tilde{a}_{\alpha,i} \to$ field $\psi^+(t, \mathbf{x})$, $V \to 0 \Rightarrow a_{\alpha,i}/V \to$ field $\psi(t, \mathbf{x})$

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- First term of the exponential

$$\sum_{\alpha} \sum_{i} \Delta t E_{+1} V \tilde{a}_{\alpha,i} \to E_{+1} \int dt \int d\mathbf{x}\, \psi^+(t,\mathbf{x})$$

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- Cumulants of second and higher order: continuum limit
  Assumption: the simplest nontrivial distribution for $\mu_\pm$ -only the variance term has a finite limit, when $\Delta t \to 0$ and $V \to 0$

$$\left(E_{+2} - E_{+1}^2\right) V\Delta t \to \sigma_+ \,, \quad \Delta t \to 0\,,\ V \to 0\,,$$
$$\left(E_{+3} - 3E_{+1}E_{+2} + E_{+1}^3\right) (V\Delta t)^2 \to 0\,, \quad \Delta t \to 0\,,\ V \to 0$$

Random sources and sinks: functional formulation

- Result: action with contribution of the average over μ_+

$$S_+ = \int dt \int d\mathbf{x} \left\{ E_{+1}\psi^+(t,\mathbf{x}) + \frac{1}{2}\sigma_+ \left[\psi^+(t,\mathbf{x})\right]^2 \right\}$$

Contribution from μ_-

$$S_- = \int dt \int d\mathbf{x} \left\{ -E_{-1}\psi^+(t,\mathbf{x})\psi(t,\mathbf{x}) + \frac{1}{2}\sigma_- \left[\psi^+(t,\mathbf{x})\psi(t,\mathbf{x})\right]^2 \right\} .$$

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- Corresponding part of action:

$$S_{1+} = \int dt \int d\mathbf{x} \left\{ E_{1+1}\psi^+\left(\psi^+ + 1\right)\psi + \frac{1}{2}\sigma_{1+}\psi^{+2}\left(\psi^+ + 1\right)^2\psi^2 \right\}$$

Scaling analysis

- $E_{+1} = \sigma_+ = 0, E_{1+1} = E_{-1}$:

  $$S_{gc} = \int dt \int d\mathbf{x} \left\{ E_{1+1} {\psi^+}^2 \psi + \frac{1}{2}\sigma_- \left(\psi^+ \psi\right)^2 + \frac{1}{2}\sigma_{1+} {\psi^+}^2 \left(\psi^+ + 1\right)^2 \psi^2 \right\}$$

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  conserves the average number of particles
- Starting point: time derivative term in the dynamic action - dimensionless

  $$S = -\int dt \int d\mathbf{x}\, \psi^+(t,\mathbf{x}) \partial_t \psi(t,\mathbf{x}) + \dots$$

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- various possibilities: Scaling dimension of ψ^+ $d_{\psi^+} = 0 \Rightarrow$ dimension of ψ $d_\psi = d \Rightarrow$ operator monomials in the second and third terms in action - the same scaling dimension - larger than that of $\psi^{+2}\psi$
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  $\Rightarrow$ are IR irrelevant $\Rightarrow$ should be discarded in the asymptotic analysis
- Scaling dimensions of both fields are positive $\Rightarrow$ above operator monomials contain ”excessive” field factor $\Rightarrow$ IR irrelevant $\Rightarrow$ should be discarded too

- IR relevant dynamic action of random sources and sinks reduces to the single term

$$S'_{gc} = \int dt \int d\mathbf{x}\, E_{1+1} \psi^{+2} \psi\,, \quad d_{\psi^+} = 0 \;\vee\; d_{\psi^+} > 0\,,\; d_\psi > 0$$

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- $d_\psi = 0 \Rightarrow d_{\psi^+} = d \Rightarrow$ (positive) $\Rightarrow$ terms with "excessive" powers of ψ^+ - IR irrelevant $\Rightarrow$ starting point for RG analysis is the source and sink action

$$S''_{gc} = \int dt \int d\mathbf{x} \left\{ E_{1+1} {\psi^+}^2 \psi + \frac{1}{2} \left(\sigma_- + \sigma_{1+}\right) \left(\psi^+ \psi\right)^2 \right\}\,, \quad d_\psi = 0$$

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Dynamic action of the diffusion-limited annihilation reaction $A + A \to \emptyset$

$$
\begin{aligned}
S_1 &= -\int_0^\infty dt \int d\mathbf{x} \left\{ \psi^+ \partial_t \psi - D_0 \psi^+ \nabla^2 \psi \right. \\
&+ \left. \lambda_0 D_0 \left[2\psi^+ + (\psi^+)^2 \right] \psi^2 \right\} + n_0 \int d\mathbf{x}\, \psi^+(\mathbf{x}, 0)
\end{aligned}
$$

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Scaling analysis: $d_{\psi^+} = 0 \Rightarrow$ scaling dimensions of nonlinear terms in action are equal
  Source-sink part of action - linear in the field ψ interaction terms of last action are quadratic in $\psi \Rightarrow$ IR irrelevant $\Rightarrow$ source-sink part of action is IR relevant

$$
\begin{aligned}
S_{IR1} &= -\int_0^\infty dt \int d\mathbf{x} \left\{ \psi^+ \partial_t \psi - D_0 \psi^+ \nabla^2 \psi - E_{1+1} {\psi^+}^2 \psi \right\} \\
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- However, this dynamic action does not bring about any graphs with closed loops of the density propagator and thus no density fluctuation effects on the asymptotic behaviour are anticipated.

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Case: $d_{\psi^+} > 0$, $d_\psi > 0$ the fourth-order term in action becomes irrelevant
  Either of the remaining third-order terms alone does not generate loops, therefore density fluctuation effects are brought about only, when both fields have the same scaling dimension $d_{\psi^+} = d_\psi = d/2$.

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- IR relevant dynamic action $\Leftrightarrow$ dynamic action of the Gribov process, also known as the Reggeon model, subject to random advection
  V.N. Gribov, Zh. Eksp. Teor. Fiz. **53**, 654 (1967).

$$
\begin{aligned}
S_{IR2} &= -\int_0^\infty dt \int d\mathbf{x} \Big\{ \psi^+ \partial_t \psi - D_0 \psi^+ \nabla^2 \psi \\
&+ 2\lambda_0 D_0 \psi^+ \psi^2 - E_{1+1} {\psi^+}^2 \psi \Big\} + n_0 \int d\mathbf{x}\, \psi^+(\mathbf{x}, 0)
\end{aligned}
$$

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Case: $d_\psi = 0$ the fourth-order term in action becomes irrelevant as well due to the positive dimension of the field ψ^+ $\wedge$ both terms of the source-sink action are also irrelevant
  $\Rightarrow$ IR relevant dynamic action

$$
\begin{aligned}
S_{IR3} &= -\int_0^\infty dt \int d\mathbf{x} \left\{ \psi^+ \partial_t \psi - D_0 \psi^+ \nabla^2 \psi + 2\lambda_0 D_0 \psi^+ \psi^2 \right\} \\
&+ n_0 \int d\mathbf{x}\, \psi^+(\mathbf{x}, 0)
\end{aligned}
$$

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\end{aligned}
$$

- This dynamic action does not give rise to any density-fluctuation loops and thus does not predict any decay anomaly due to them.
- Lesson: If the sources and sinks are chosen such that they conserve the mean number of particles in the system, then the anomalous scaling behaviour in the system is that of the Gribov process, if any.

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Case: plain source term is not vanished $\Rightarrow$ there is the possibility that the system does not tend to the absorbing empty state but to an active state with a finite concentration of particles.

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

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- Starting point: dynamic action with all the terms quoted above

$$S = \int_0^\infty dt \int d\mathbf{x} \Bigg\{ -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi$$

$$-\lambda_0 D_0 \left[2\psi^+ + \left(\psi^+\right)^2 \right] \psi^2 + E_{+1} \psi^+ + \frac{1}{2}\sigma_+ \left(\psi^+\right)^2 + E_{1+1} \psi^+ \left(\psi^+ + 1\right) \psi$$

$$+ \frac{1}{2}\sigma_{1+} \psi^{+2} \left(\psi^+ + 1\right)^2 \psi^2 - E_{-1} \psi^+ \psi + \frac{1}{2}\sigma_- \left(\psi^+ \psi\right)^2 \Bigg\} + n_0 \int d\mathbf{x}\, \psi^+(\mathbf{x}, 0)$$

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- Stationarity equation brought about by this dynamic action for the field ψ:

$$\partial_t \psi - D_0 \nabla^2 \psi = -2\lambda_0 D_0 \psi^2 + E_{+1} + E_{1+1} \psi - E_{-1} \psi$$

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Action expanded around the stationary value ($E_{1+1} = E_{-1}$):

$$S = \int_0^\infty dt \int d\mathbf{x} \Big\{ -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi - \sqrt{8}\sqrt{E_{+1}\lambda_0 D_0}\psi^+\psi$$

$$+ \left(\frac{-E_{+1}}{2} + \frac{E_{-1}\sqrt{E_{+1}\,\lambda_0 D_0}}{\sqrt{2}\lambda_0 D_0} + \frac{E_{+1}\sigma_{1+}}{4\lambda_0 D_0} + \frac{E_{+1}\sigma_{-}}{4\lambda_0 D_0} + \frac{\sigma_+}{2} \right) \psi^{+2}$$

$$+ \frac{E_{+1}\sigma_{1+}\psi^{+3}}{2\lambda_0 D_0} + \frac{E_{+1}\sigma_{1+}\psi^{+4}}{4\lambda_0 D_0} + \frac{\sqrt{2}\,\sqrt{E_{+1}\,\lambda_0 D_0}\,\sigma_{1+}\,\psi^{+3}\psi}{\lambda_0 D_0}$$

$$+ \left(E_{-1} - \sqrt{2}\,\sqrt{E_{+1}\,\lambda_0 D_0} + \frac{\sqrt{E_{+1}\,\lambda_0 D_0}\,\sigma_{1+}}{\sqrt{2}\,\lambda_0 D_0} + \frac{\sqrt{E_{+1}\,\lambda_0 D_0}\,\sigma_{-}}{\sqrt{2}\,\lambda_0 D_0} \right) \psi^{+2}\psi$$

$$+ \frac{\sqrt{E_{+1}\,\lambda_0 D_0}\,\sigma_{1+}\,\psi^{+4}\psi}{\sqrt{2}\,\lambda_0 D_0} - 2\,\lambda_0 D_0\,\psi^+\psi^2 + \left(-\lambda_0 D_0 + \frac{\sigma_{1+}}{2} + \frac{\sigma_{-}}{2} \right) \psi^{+2}\psi^2$$

$$+ \sigma_{1+}\,\psi^{+3}\psi^2 + \frac{\sigma_{1+}\,\psi^{+4}\psi^2}{2} \Big\} + n_0 \int d\mathbf{x}\,\psi^+(\mathbf{x}, 0)\,.$$

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Critical limit $E_{+1} \to 0 \Rightarrow$ variance σ_+ vanishes (expectation value of a nonnegative random quantity μ_+) $\Rightarrow$ Vicinity of the critical point - only the leading E_{+1} and σ_+ terms

$$
\begin{aligned}
S = \int_0^\infty dt \int d\mathbf{x} \Big\{ & -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi - \sqrt{8}\sqrt{E_{+1}\lambda_0 D_0}\psi^+\psi \\
& + \left(\frac{E_{-1}\sqrt{E_{+1}}}{\sqrt{2\lambda_0 D_0}} + \frac{\sigma_+}{2} \right) \psi^{+2} + \frac{E_{+1}\,\sigma_{1+}\,\psi^{+3}}{2\,\lambda_0 D_0} + \frac{E_{+1}\,\sigma_{1+}\,\psi^{+4}}{4\,\lambda_0 D_0} + E_{-1}\,\psi^{+2}\psi \\
& + \frac{\sqrt{2}\sqrt{E_{+1}\,\lambda_0 D_0}\,\sigma_{1+}\,\psi^{+3}\psi}{\lambda_0 D_0} + \frac{\sqrt{E_{+1}\,\lambda_0 D_0}\,\sigma_{1+}\,\psi^{+4}\psi}{\sqrt{2}\,\lambda_0 D_0} - 2\,\lambda_0 D_0\,\psi^+\psi^2 \\
& + \left(-\lambda_0 D_0 + \frac{\sigma_{1+}}{2} + \frac{\sigma_-}{2} \right) \psi^{+2}\psi^2 + \sigma_{1+}\,\psi^{+3}\psi^2 + \frac{\sigma_{1+}\,\psi^{+4}\psi^2}{2} \Big\} + n_0 \int d\mathbf{x}\,\psi^+
\end{aligned}
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Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Dimensional analysis: Nonlinear parts without the critical parameters E_{+1} and σ_+ - previous arguments hold

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  canonical dimension of E_{+1} is four (from quadratic-field part of the action)
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- terms having powers of these parameters as coefficients: their positive scaling dimensions must be taken into account
  canonical dimension of E_{+1} is four (from quadratic-field part of the action)
  canonical dimension of σ_+ remains a free parameter
- Case with $d_{\psi^+} = 0$: the third and fourth powers of ψ^+ and terms independent of ψ or first order in ψ are irrelevant compared with terms $\propto \psi^{+2}$ due to the coefficients proportional to E_{+1} or its square root
  Nonlinear in ψ terms are irrelevant against the linear terms due to positive dimension of ψ

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- IR effective action

$$
\begin{aligned}
S &= \int_0^\infty dt \int d\mathbf{x} \Big\{ -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi \\
&- 2\sqrt{2}\sqrt{E_{+1}\,\lambda_0 D_0}\,\psi^+\psi + \left(\frac{E_{-1}\sqrt{E_{+1}}}{\sqrt{2\lambda_0 D_0}} + \frac{\sigma_+}{2} \right) {\psi^+}^2 \\
&+ E_{-1}\,{\psi^+}^2\psi \Big\} + n_0 \int d\mathbf{x}\,\psi^+(\mathbf{x},0)
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&+ E_{-1}\,\psi^{+2}\psi \Big\} + n_0 \int d\mathbf{x}\,\psi^+(\mathbf{x}, 0)
\end{aligned}
$$

- Conclusion: remaining interaction terms do not induce loops (although here we have a nontrivial correlation function of the field ψ)

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Case with $d_{\psi^+} > 0$ and $d_\psi > 0$: higher powers than the leading corrections to the quadratic-field action of both fields are irrelevant
  This argument leaves us with the dynamic action

$$S = \int_0^\infty dt \int d\mathbf{x} \Big\{ -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi - \sqrt{8}\sqrt{E_{+1}\lambda_0 D_0}\psi^+ \psi$$
$$+ \left(\frac{E_{-1}\sqrt{E_{+1}}}{\sqrt{2\lambda_0 D_0}} + \frac{\sigma_+}{2} \right) {\psi^+}^2 + E_{-1} {\psi^+}^2 \psi - 2\lambda_0 D_0 \psi^+ \psi^2 \Big\} + n_0 \int d\mathbf{x} \psi^+(\mathbf{x}, 0)$$

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- Result: Interaction term $-2\,\lambda_0 D_0\,\psi^+\psi^2$ generates loops alone due to the presence of the correlation function of the field $\psi \Rightarrow$ two effective actions with nontrivial fluctuation contributions are possible

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- a) $d_{\psi^+} > d_\psi$: to keep the correlation function of the field ψ for the loops, the variance σ_+ must have a dimension less than that dim. of $\sqrt{E_{+1}}$
Effective action

$$S = \int_0^\infty dt \int d\mathbf{x} \Big\{ -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi - \sqrt{8} \sqrt{E_{+1} \lambda_0 D_0} \psi^+ \psi + \frac{\sigma_+}{2} \psi^{+2} - 2\,\lambda_0 D_0\, \psi^+ \psi^2 \Big\} + n_0 \int d\mathbf{x}\, \psi^+(\mathbf{x}, 0)$$

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- Model is logarithmic at dimension $d = 6$
  apart from the coefficient of the $\propto {\psi^+}^2$ the action is that of critical dynamics of the φ^3 model

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- b) $d_{\psi^+} = d_\psi = d/2$: Both third-order terms are relevant
  dimension of σ_+ - larger than that of $\sqrt{E_{+1}} \Rightarrow \sigma_+$ can be omitted
  Effective dynamic action

$$S = \int_0^\infty dt \int d\mathbf{x} \Big\{ -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi - \sqrt{8}\sqrt{E_{+1}\lambda_0 D_0}\psi^+ \psi + \frac{E_{-1}\sqrt{E_{+1}}}{\sqrt{2\lambda_0 D_0}}\psi^{+2} + E_{-1}\psi^{+2}\psi - 2\lambda_0 D_0 \psi^+ \psi^2 \Big\} + n_0 \int d\mathbf{x}\, \psi^+(\mathbf{x}, 0)$$

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- c) dimension of σ_+ - smaller than that of $\sqrt{E_{+1}} \Rightarrow$ "mass term"can be omitted
  Effective dynamic action

$$S = \int_0^\infty dt \int d\mathbf{x} \Big\{ -\psi^+ \partial_t \psi + D_0 \psi^+ \nabla^2 \psi + \frac{\sigma_+}{2}\psi^{+2} + E_{-1}\psi^{+2}\psi - 2\lambda_0 D_0 \psi^+\psi^2 \Big\} + n_0 \int d\mathbf{x}\, \psi^+(\mathbf{x},0)$$

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Dynamic actions describing the Gribov process with a random source include terms independent of the active agent density, which are not present in the Langevin approach!!!

Annihilation reaction $A + A \to \emptyset$ with random sources and sinks

- Dynamic actions describing the Gribov process with a random source include terms independent of the active agent density, which are not present in the Langevin approach!!!
- Indeed, the fact that the rate of change of the density due to the random sink is proportional to a power of density is a natural assumption.
  The assumption that the rate of change of the density due to the random source is proportional to a power of density is not natural.
  Therefore, the dynamic action in cases b) and c) possibly predicts a critical behaviour of the Gribov process different from that discussed in the literature.

Conclusions

- we have investigated possible effects of sources and sinks in reaction kinetics. They have been introduced directly in the master equation.

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  Most of the dynamic effective actions obtained are trivial from the point of view of critical behaviour of the model, because infrared relevant interaction terms do not generate loops diagrams.

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- Non-trivial cases lead to the action describing Gribov process. However, in this manner we obtain a source term in dynamic action without agent density. This is cardinally different from that generated by Langevin equation.

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- Non-trivial cases lead to the action describing Gribov process. However, in this manner we obtain a source term in dynamic action without agent density. This is cardinally different from that generated by Langevin equation.
- For the study of influence of density fluctuations on annihilation reaction $A + A \to \emptyset$, which are relevant only for dimension two or less, random sources and sinks are unimportant.