

The right relativity for QFT

- Galilean relativity
- Special relativity
- Double (deformed) special relativity
- de Sitter relativity
- Anti-de Sitter special relativity
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Waldemar von Ignatowsky

graduated in 1906 at the Petersburg University. In 1906-1908 he continued to study at the Giessen University, with his dissertation in 1909. In 1911-1914 he taught at the Higher Technical School in Berlin. During WWI he worked at “Karl Zeiss” near Paris (Orsay). After February revolution in Russia he returned to Petrograd to work at the future State Optical Institute. Here he taught Electrodynamics at the Petrograd (Leningrad) university. He became an associated member of the Academy of Sciences in 1932 г. (applied optics). Solzhenitsyn reported in “Arhipelag gulag”, that Ignatowski was arrested by NKVD as German spy and executed in 1942 in Leningrad.

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ZUR WELLENGLEICHUNG IM N -DIMENSIONALEN EUKLIDISCHEN
RAUM

Von V. IGNATOVSKIJ (W. v. Ignatowsky)

(Présenté par A. Krylov, membre de l'Académie des Sciences)

Die Wellengleichnung im n -dimensionalen Raum lautet:

$$(1) \quad \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} - \sum_{i=1}^{i=n} \frac{\partial^2 u}{\partial x_i^2} = 0,$$

wobei t die Zeit, c die Geschwindigkeit und x_i die rechtwinkligen Koordinaten im n -dimensionalen Euklidischen Raum bedeuten.

W. v. Ignatowsky (Berlin), Einige allgemeine Bemerkungen zum Relativitätsprinzip.

Als Einstein seinerzeit das Relativitätsprinzip einführte, nahm er parallel mit demselben an, daß die Lichtgeschwindigkeit c eine universelle Konstante sei, d.h. für alle Koordinatensysteme denselben Wert behalte. Auch Minkowski ging bei seinen Untersuchungen von der Invariante $r^2 - c^2 t^2$ aus, obwohl nach seinem Vortrage „Raum und Zeit“¹⁾ zu urteilen, er dem c mehr die Bedeutung einer universellen Raum-Zeit-Konstante beilegte, als diejenige der Lichtgeschwindigkeit.

Nun habe ich mir die Frage gestellt, zu welchen Beziehungen bezw. Transformationsgleichungen man kommt, wenn man nur das Relativitätsprinzip an die Spitze der Untersuchung stellt und ob überhaupt die Lorentzschen Transformationsgleichungen die einzigen sind, die dem Relativitätsprinzip genügen.

In 1910 Ignatowsky was first who tried to apply group theory to the derivation of Lorentz transformations without the postulate of the constancy of the speed of light.

Instead of supposition: $c' = c$

he required: if $K(u)$ and $K'(u')$ are two transformations, then

$$K(u) K'(u') = K''(u'')$$

V. A. Fock, Theory of space, time and gravitation, Appendix A:

Мы приходим к следующему выводу. Самый общий вид преобразования (A.01), удовлетворяющего условию (а), есть преобразование при помощи дробно-линейных функций с одним и тем же знаменателем. В том частном случае, когда знаменатель приводится к постоянной, дробные функции приводятся к целым.

The requirement of the linearity is equivalent to the speed of light constancy postulate

W. v. Ignatowsky, Physik. Zeitschr. XI, p. 972 (1910)

Wir nehmen jetzt in K und K' zwei Elemente dx und dx' von solcher Länge, daß, wenn sie gegenseitig auf Ruhe gebracht würden, sie gleich lang sind. Messen wir jetzt dx' synchron von K aus (also $dt = 0$), so erhalten wir
$$dx' = \beta dx. \quad (13)$$

Messen wir dx synchron von K' aus (also $dt' = 0$), so folgt dementsprechend

$$dx = \beta' dx'. \quad (14)$$

Возьмем теперь в K и K' два элемента dx и dx' такой длины, что, если их привести в состояние взаимного покоя, она будет одинаковой. Если мы измерим теперь dx' синхронно в K (таким образом, $dt = 0$), то получим: $dx' = pdx$.

Если мы измерим dx синхронно в K' (таким образом, $dt' = 0$), то, соответственно $dx = p'dx'$.

The usual a priori assumption that all bodies at rest in K' have the same velocity \mathbf{u} in K is equivalent to linearity of the transformation.

This assumption is not valid in the case of linear fractional transformations.

So, we had to be careful in the definition of the relative velocities.

Let us introduce coordinates in such a way that point $\mathbf{r} = 0$ at rest in frame K has constant velocity $-\mathbf{u}$ in K' ; point $\mathbf{r}' = 0$ at rest in frame K' has constant velocity \mathbf{u} in K .

We take that at $t = 0$ the spatial origins of both frames coincide and $t = 0$.

Fractionally linear transformations, compatible with rotations

$$t' = \frac{a(u)t + b(u)(\mathbf{u}\mathbf{r})}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})},$$

$$\mathbf{r}_{\parallel}' = \frac{d(u)\mathbf{u} + e(u)\mathbf{r}_{\parallel}}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})},$$

$$\mathbf{r}_{\perp}' = \frac{f(u)\mathbf{r}_{\perp}}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})}.$$

Result

$$t' = \gamma \frac{t - \frac{(\mathbf{u}\mathbf{r})}{c_0^2}}{1 - (\gamma - 1) \frac{c_0 t}{R} + \gamma \frac{(\mathbf{u}\mathbf{r})}{c_0 R}},$$

$$\mathbf{r}_{\parallel}' = \gamma \frac{\mathbf{r}_{\parallel} - \mathbf{u}t}{1 - (\gamma - 1) \frac{c_0 t}{R} + \gamma \frac{(\mathbf{u}\mathbf{r})}{c_0 R}},$$

$$\mathbf{r}_{\perp}' = \frac{\mathbf{r}_{\perp}}{1 - (\gamma - 1) \frac{c_0 t}{R} + \gamma \frac{(\mathbf{u}\mathbf{r})}{c_0 R}},$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c_0^2}}}$$

$$t = -\frac{R}{c_0} \Leftrightarrow t' = -\frac{R}{c_0}$$

$$t \rightarrow t - \frac{R}{c_0} \equiv t - t_0$$

**Fock-Lorentz transformations (FLT)
or Fractionally linear transformations**

$$t' = \frac{t}{\gamma - (\gamma - 1)t/t_0 + \gamma(\mathbf{u}\mathbf{r})t_0/R^2},$$

$$\mathbf{r}_{\parallel}' = \gamma \frac{\mathbf{r}_{\parallel} - \mathbf{u}(t - t_0)}{\gamma - (\gamma - 1)t/t_0 + \gamma(\mathbf{u}\mathbf{r})t_0/R^2}, \quad t_0 = \frac{R}{c}$$

$$\mathbf{r}_{\perp}' = \frac{\mathbf{r}_{\perp}}{\gamma - (\gamma - 1)t/t_0 + \gamma(\mathbf{u}\mathbf{r})t_0/R^2}$$

$$R = \text{const}, \quad c_0 = \text{const} \quad \text{How it may be?} \quad c_0 = \frac{R}{t_0}$$

1. Galilean transformations

$$t' = t; \quad \mathbf{r}' = \mathbf{r} - \mathbf{u}t$$

2. Special translations

$$t'' = \frac{t' \sqrt{1 - \frac{\rho^2}{R^2}}}{1 - \frac{(\mathbf{r}' \cdot \mathbf{\rho})}{R^2}}; \quad \mathbf{r}''_{||} = \frac{\mathbf{r}'_{||} - \mathbf{\rho}}{1 - \frac{(\mathbf{r}' \cdot \mathbf{\rho})}{R^2}}, \quad \mathbf{\rho} = -\mathbf{u}t_0$$

3. Inverse time translations

$$t''' = \frac{t''}{1 + \frac{\tau t''}{t_0^2}}; \quad \mathbf{r}''' = \frac{\mathbf{r}''}{1 + \frac{\tau t''}{t_0^2}}, \quad \tau = t_0(1 - \gamma^{-1}).$$

Galilean symmetry

$$\mathbf{r}' = \mathbf{r} - \mathbf{p}, \quad \mathbf{u}' = \mathbf{u} - \mathbf{v},$$

Relativistic space-time symmetry (Poincaré) as deformation of Galilean symmetry

$$\mathbf{r}' = \mathbf{r} - \mathbf{p}, \quad \mathbf{u}_{\parallel}' = \frac{\mathbf{u}_{\parallel} - \mathbf{v}}{1 - \frac{\mathbf{u}_{\parallel} \cdot \mathbf{v}}{c^2}}, \quad \mathbf{u}'_{\perp} = \frac{\mathbf{u}_{\perp} \sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}{1 - \frac{\mathbf{u}_{\parallel} \cdot \mathbf{v}}{c^2}}$$

Second (Fock-Lorentz) deformation of Galilean symmetry

$$\mathbf{r}'_{\parallel} = \frac{\mathbf{r}_{\parallel} - \mathbf{p}}{1 - \frac{\mathbf{r}_{\parallel} \cdot \mathbf{p}}{R^2}}, \quad \mathbf{r}'_{\perp} = \frac{\mathbf{r}_{\perp} \sqrt{1 - \frac{\rho^2}{R^2}}}{1 - \frac{\mathbf{r}_{\parallel} \cdot \mathbf{p}}{R^2}}, \quad t' = \frac{t \sqrt{1 - \frac{\rho^2}{R^2}}}{1 - \frac{\mathbf{r}_{\parallel} \cdot \mathbf{p}}{R^2}}, \quad \mathbf{u}' = \mathbf{u} - \mathbf{v},$$

Combined (Anti-de Sitter) deformation of Galilean symmetry

$$ct' = \frac{ct - \mathbf{v}\mathbf{r}}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}; \quad \mathbf{r}_{\parallel}' = \frac{\mathbf{r}_{\parallel} - \mathbf{v}t}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}; \quad \text{Lorentz}$$

$$x'^{\mu} = \frac{x^{\mu} - \alpha^{\mu} + (\gamma - 1) \frac{\alpha^{\mu}}{\alpha^2} [(\alpha x) - \alpha^2]}{\gamma \left(1 + \frac{(\alpha x)}{R^2} \right)} \quad \text{4-dim “translations”}$$

$$x^{\mu} = (ct, \mathbf{r}); \quad a^{\mu} = (c\theta, \mathbf{p}); \quad (ax) = c^2 \theta t - \mathbf{p}\mathbf{r}; \quad \gamma = \frac{1}{\sqrt{1 + \frac{\alpha^2}{R^2}}}$$

4-dim translations in 3-dim form

$$\theta = 0 \quad \quad \quad t' = \frac{t\sqrt{1-\frac{\rho^2}{R^2}}}{1-\frac{(\mathbf{r}\rho)}{R^2}}; \quad \quad \quad \mathbf{r}'_{||} = \frac{\mathbf{r}_{||}-\mathbf{\rho}}{1-\frac{(\mathbf{r}\rho)}{R^2}},$$

$$\rho = 0 \quad \quad \quad t' = \frac{t'-\theta}{1+\frac{c^2\theta t}{R^2}}; \quad \quad \quad \mathbf{r}' = \frac{\mathbf{r}\sqrt{1+\frac{c^2\theta^2}{R^2}}}{1+\frac{c^2\theta t}{R^2}},$$

$$c\rightarrow\infty,\qquad \theta=\tau\frac{R^2}{c^2t_0^{-2}}$$

Fock-Lorentz mechanics

This form is invariant with respect to FLT:

$$ds^2 = \frac{R^4}{c_0^2 t^4} dt^2 \left(1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} \right); \quad \mathbf{v} = d\mathbf{r}/dt$$

FL action for free particle with mass $m \Leftrightarrow$ Lorentzian action

$$S = -mc_0 \int ds = -m \int \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt \Leftrightarrow S = -m \int c_0^2 \sqrt{1 - \frac{\mathbf{v}^2}{c_0^2}} dt$$

$$t = t_0 + \tau; \quad \tau \ll t_0; \quad |\mathbf{r}| \ll R$$

Conserved quantities for FL free particles

$$S = -m \int \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

$$H = \frac{mR^2}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}} \quad \mathbf{K} = \frac{m(\mathbf{v}t - \mathbf{r})}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}}$$

$$H^2 - \mathbf{K}^2 R^2 = m^2 R^4$$

$$\mathbf{J} = \frac{m(\mathbf{r} \times \mathbf{v})}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}} \quad \mathbf{P} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}}$$

Anti-de Sitter Action in Beltrami coordinates

$$S = -mc \int ds = -m \int \frac{c^2 R^2}{(R^2 + c^2 t^2 - \mathbf{r}^2)} \sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}} dt.$$

$$\frac{1}{c_0^2} = \frac{1}{c^2} + \frac{t_0^2}{R^2}$$

Nonrelativistic limit

$$c \rightarrow \infty$$

Noncosmological limit

$$R \rightarrow \infty$$

$$S = -m \int \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

$$S = -m \int c^2 \sqrt{1 - \frac{\mathbf{v}^2}{c^2}} dt$$

Conserved quantities for Anti-de Sitter free particles

$$H = \frac{m}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}};$$

$$\mathbf{P} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}};$$

$$\mathbf{K} = \frac{m(\mathbf{v}t - \mathbf{r})}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}};$$

$$\mathbf{J} = \frac{m[\mathbf{v} \times (\mathbf{v}t - \mathbf{r})]}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}}}$$

$$H^2 - \frac{\mathbf{K}^2}{R^2} - \frac{\mathbf{P}^2}{c^2} + \frac{\mathbf{J}^2}{R^2 c^2} = m^2$$

$$\mathbf{J} = \frac{[\mathbf{P} \times \mathbf{K}]}{H}$$

FL energy in noncosmological limit

$$H = \sum \frac{mR^2}{\sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}}} \quad |\mathbf{v}t_0 - \mathbf{r}| \ll R$$

$$H = \sum mR^2 \left(1 + \frac{(\mathbf{v}(t_0 + \tau) - \mathbf{r})^2}{2R^2} - \dots \right) \quad t = t_0 + \tau; \quad \tau \ll t_0$$

$$H = \sum mR^2 + t_0^2 \sum m \frac{\mathbf{v}^2}{2} + t_0 \sum m \mathbf{v}(\mathbf{v}\tau - \mathbf{r}) + \sum m \frac{(\mathbf{v}\tau - \mathbf{r})^2}{2}$$

$$\text{Let } \frac{\partial H}{\partial t_0} = 0$$

$$E = \sum m \frac{\mathbf{v}^2}{2}; \quad I = \sum m \frac{(\mathbf{v}\tau - \mathbf{r})^2}{2}; \quad F = \sum m \mathbf{v}(\mathbf{v}\tau - \mathbf{r}).$$

“Two” Galilean relativity are identical

$$S = -mc \int ds = -m \int \frac{c^2 R^2}{(R^2 + c^2 t^2 - \mathbf{r}^2)} \sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}} dt.$$

$$c \rightarrow \infty$$

$$R \rightarrow \infty$$

$$S = -m \int \frac{R^2}{t^2} \sqrt{1 - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2}} dt$$

$$S = -m \int c^2 \sqrt{1 - \frac{\mathbf{v}^2}{c^2}} dt$$

$$R \rightarrow \infty$$

$$c \rightarrow \infty$$

$$S = m \int \frac{(\mathbf{v}t - \mathbf{r})^2}{2t^2} dt$$

$$S = m \int \frac{\mathbf{v}^2}{2} dt$$

Nonrelativistic free motion symmetry: $S = \int \sum \frac{m\mathbf{v}^2}{2} dt$

1. 3 rotations
2. 3 Galilean boosts
3. 3 Galilean translations
4. time translation
5. inverse time translation:

$$\frac{1}{t'} = \frac{1}{t} + \alpha; \quad \frac{\mathbf{r}'}{t'} = \frac{\mathbf{r}}{t}$$

$$I = \sum m \frac{(\mathbf{v}\tau - \mathbf{r})^2}{2} = \sum \frac{\mathbf{K}^2}{2m}$$

5. dilatation:

$$t' = \alpha^2 t$$

$$\mathbf{r}' \rightarrow \alpha \mathbf{r}$$

$$F = \sum m\mathbf{v}(\mathbf{v}\tau - \mathbf{r})$$

Anti-de Sitter Action in Beltrami coordinates

$$S = -mc \int ds = -m \int \frac{c^2 R^2}{(R^2 + c^2 t^2 - \mathbf{r}^2)} \sqrt{1 - \frac{\mathbf{v}^2}{c^2} - \frac{(\mathbf{v}t - \mathbf{r})^2}{R^2} + \frac{(\mathbf{r} \times \mathbf{v})^2}{R^2 c^2}} dt.$$

Nonrelativistic limit $c \rightarrow \infty$

$$ct \gg R, \quad t \gg \frac{R}{c} = \tau?$$

$$\tau = \sqrt{\frac{\hbar G_N}{c^5}} = 5.39 \cdot 10^{-44} \text{ s}$$

$$t_0 = 10^{60} \tau$$