Low Dimensional Yang-Mills: Matrix Models and Emergent Geometry

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Motivation

- Is the dimensionality of spacetime fixed or dynamical?
- Are spacetime geometry and topology inputs or outputs of the dynamics?

It is now possible to make models where these are well posed questions and with spacetime emerging from more primitive structures.

More generally these are questions for quantum gravity. However,

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The results presented below are based on:

- R. Delgadillo-Blando, Denjoe O'Connor and B. Ydri, Phys. Rev. Lett. 100,201601 (2008) [arXiv:0712.3011]
- R. Delgadillo-Blando, Denjoe O'Connor and B. Ydri, JHEP05 (2009) 049 [arXiv:0806.0558]

Yang-Mills with adjoint Fermions

$$S[A,\Psi] = \int d^d x Tr(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\psi^T C\Gamma^\mu D_\mu \psi)$$

A_μ and ψ are in the adjoint representation of the gauge group.
Dimensional reduction by taking the fields independent of p co-oridnates gives interesting models.
The most drastic reduction is to reduce to zero dimensions ($p = d$)

For $d = 10$ reduction to zero dimensions $\Longrightarrow$ IKKT matrix model.
Reduction to $d = 1$ (so that $p = 9$) gives the BFSS matrix model.
And reduction to $d = 4$ (i.e. $p = 6$) gives $\mathcal{N} = 4$ susy.

The zero-dimensional model with d $N \times N$-matrices:

$$S(X,\psi) = NTr(-\frac{1}{4}[X_\mu, X_\nu]^2 + \psi^T \Gamma^\mu [X_\mu, \psi]) \qquad (1)$$

For $d = 4$ the model reduces to a the 2-matrix model

$$\mathcal{Z} = \int [dX][dY] e^{-\mathrm{tr} X^2 - \mathrm{tr} Y^2 + g^2 \mathrm{tr}[X,Y]^2} \ . \qquad (2)$$

This in turn can be expressed as a the 3-matrix model.

$$\tilde{\mathcal{Z}} = \int [dX][dY][dZ] e^{-\mathrm{tr} X^2 - \mathrm{tr} Y^2 - \mathrm{tr} Z^2 + i\alpha \mathrm{tr}[X,Y]Z} \ . \qquad (3)$$

This is just a Gaussian model with Myers term. Setting $g^2 = (i\alpha)^2/4$ gives the equivalence of the models.

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We will consider

$$S(X) = NTr(-\frac{1}{4}[X_a, X_b]^2 + \frac{2i\alpha}{3}\epsilon_{abc}X_aX_bX_c)$$

Rescale $X_a = \alpha D_a$ to get $S(X) = \beta E(D)$ where $\beta = \alpha^4 N^2 = \tilde{\alpha}^4$.

$$E(D) = \frac{Tr}{N}(-\frac{1}{4}[D_a, D_b]^2 + \frac{2i\alpha}{3}\epsilon_{abc}D_aD_bD_c)$$

A 3-matrix model with $SO(3)$ symmetry.

The most general quartic single trace 3-matrix model with global $SO(3)$ symmetry has energy

$$E = \frac{Tr}{N}\left(-\frac{1}{4}[D_j, D_k]^2 + \frac{2\mathrm{i}}{3}\epsilon_{jkl} D_j D_k D_l + bD_j^2 + c(D_j^2)^2\right)$$

The Potential $V(D) = Tr(bD_j^2 + c(D_j^2)^2)$
breaks $D_j \to D_j + d_j\mathbf{1}$ symmetry.

Partition Function

$$Z(\beta, g, b, c) = \int [dD_j] \mathrm{e}^{-S(D)} \qquad \text{where} \qquad S(D) = -\beta E(D)$$

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The critical points of the model with $V = 0$ are given by

$$[D_k, ([D_j, D_k] - \mathrm{i}\epsilon_{jkl} D_l)] = 0.$$

So representations of the Lie algebra of $SU(2)$ are critical points with energy $E_{saddle} = -\frac{1}{6}\frac{Tr}{N}(D_j^2)$.

The minimum energy configuration is

$D_j = L_j$ with $E_0 = -\frac{N^2-1}{24}$.

The L_j satisfy

$[L_j, L_j] = \mathrm{i}\epsilon_{jkl} L_l$ and $L_j L_j = \frac{N^2-1}{4}\mathbf{1}$.

These are the familiar commutation relations of angular momentum.

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Geometry

Consider the Dirac operator

$$\mathcal{D} = \sigma_j[D_j, \cdot] + 1 \ ,$$

with $\mathcal{D}\Psi = \sigma_j[D_j, \Psi] + \Psi$.
Then one can see the ground state geometry via the "spectral triple" $(\mathcal{H}, Mat_N, \mathcal{D}_0)$, where the algebra is Mat_N with trace norm and

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This Dirac operator has the same spectrum as that of the commutative sphere but with a cutoff at high energies.
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A sphere from matrices

Let $N_j = \frac{2}{\sqrt{N^2-1}} L_j$

We get a sphere

$N_1^2 + N_2^2 + N_3^2 = \mathbf{1}$.   A nice round sphere.

But it is non-commutative.

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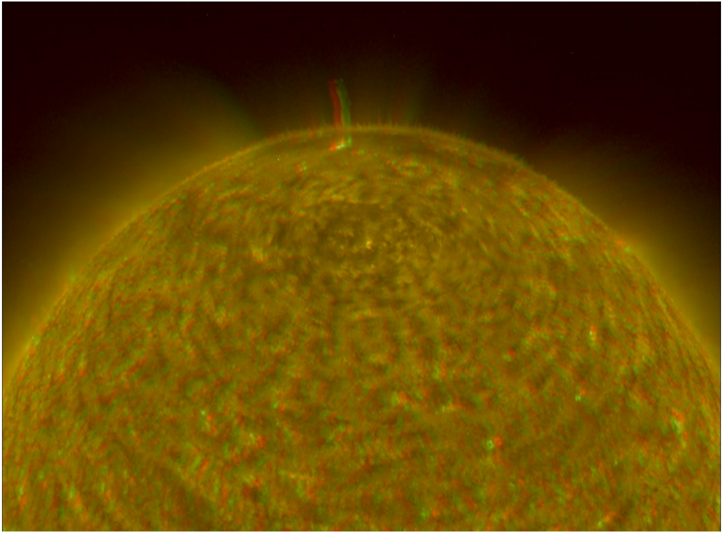
Our "fuzzy" sphere



A fuzzy field theory model.

Fuzzy field theories are matrix models with fixed background matrices. The scalar field theory of the fuzzy sphere has:
$S_N(\Phi, a, b, c) = Tr(-a[L_j, \Phi]^2 + b\Phi^2 + c\Phi^4)$
L_j are the generators of $su(2)$ in the N dimensional representation.
again with Φ an $N \times N$ matrix.

The action $S_N(\Phi, a, b, c)$ converges for $N \to \infty$ to the action of a scalar field ϕ on the round commutative sphere.

$$\lim_{N\to\infty} \left| S(\phi, r, \lambda) - S_N(\Phi, \frac{1}{2N}, \frac{r}{2N}, \frac{\lambda}{4!N}) \right| \to 0 \, .$$

The matrix $\Phi = \int_{S^2} \omega \rho_N \phi$, with ω the unit volume form on S^2 and ρ_N is a particular matrix valued function on S^2.

$$\rho_N = \sum_{lm} Y_{lm} \hat{Y}_{lm}$$

where Y_{lm} are the spherical harmonics and $\hat{Y}_{lm}$ are polarization tensors satisfying

$$[L_3, \hat{Y}_{lm}] = m\hat{Y}_{lm} \qquad \text{and} \qquad [L_j, [L_j, \hat{Y}_{lm}] = l(l+1)\hat{Y}_{lm} \ .$$

So that if

$$\phi = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{lm} Y_{lm}$$

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Small fluctuations

The zero temperature ground state of the model
$E = \frac{Tr}{N}(-\frac{1}{4}[D_j, D_k]^2 + \frac{2\mathrm{i}}{3}\epsilon_{jkl}D_jD_kD_l)$
is a round fuzzy sphere with $D_j = L_j$ and $E_0 = -\frac{L_j^2}{6}$.

Expanding around the minimum solution, $D_j = L_j + A_j$ yields a noncommutative Yang-Mills action with field strength
$F_{jk} = i[L_j, A_k] - i[L_j, A_k] + \epsilon_{jkl}A_l + i[A_j, A_bk]$.
As written the gauge field includes a scalar field,

$$\Phi = \frac{1}{\sqrt{N^2-1}}(D_j - L_j)^2 = \frac{1}{2}(N_jA_j + A_jN_j + \frac{A_j^2}{\sqrt{c_2}}) .$$

It is the component of the gauge field normal to the sphere when viewed as imbeded in $\mathbf{R}^3$ with $N_j = \frac{L_j}{\sqrt{c_2}}$ and $c_2 = L_j^2 = (N^2-1)/4$.

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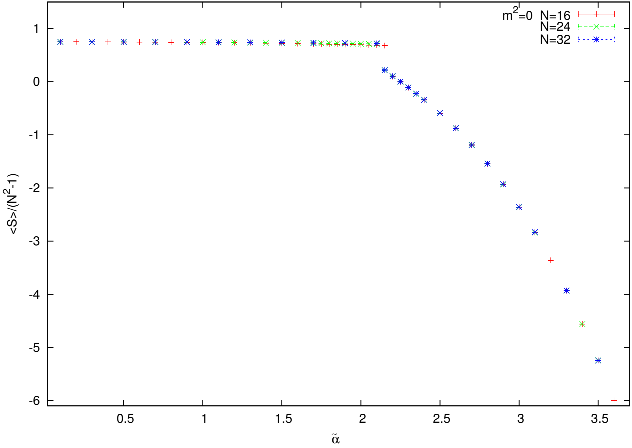
Increasing the temperature. Monte Carlo Simulations

Defining $\mathcal{S} = < S >$ and $\beta = \tilde{\alpha}^4$



The entropy jump

$\mathcal{S} = \frac{5}{12}$ as the transition is approached from the fuzzy sphere side,

and jumps to $\mathcal{S} = \frac{3}{4}$ in the high temperature phase.

The infinite temperature entropy does not contribute $\frac{1}{2}$ but $\frac{1}{4}$ per degree of freedom.

So the model remains highly interacting at high temperatures.

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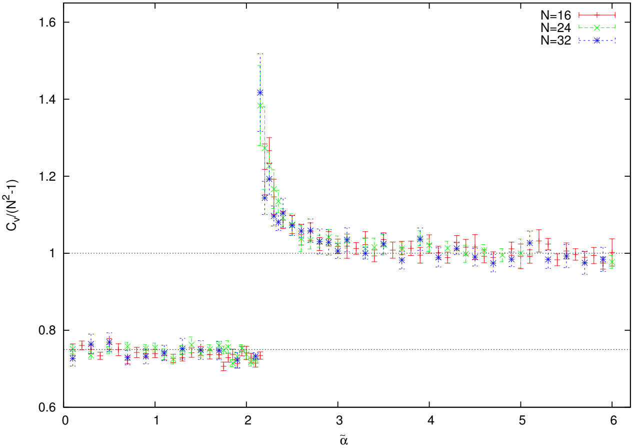
Specific Heat

The specific heat C_v/N^2 where $C_v = \langle S^2 \rangle - \langle S \rangle^2$ and



$\beta = \tilde{\alpha}^4$

Specific Heat Exponent

Entropy Jump

The transition is unusual in that it has a jump in the entropy. $\Delta\mathcal{S} = \frac{1}{3}$ indicating a 1st order transition.

Divergent Specific Heat

But it has a divergent specific heat $C = A_{-}(T_c - T)^{-\alpha}$ typical of a continuous (or second order) transition. We find the specific heat exponent $\alpha = \frac{1}{2}$.

Our analysis gives the critical point $\beta_c = (\frac{8}{3})^3$ and a critical exponent $\alpha = \frac{1}{2}$ for the divergence of the specific heat.

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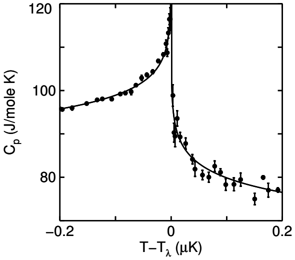
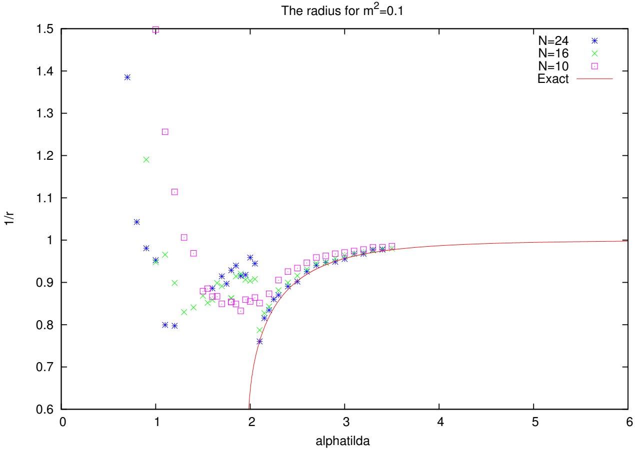
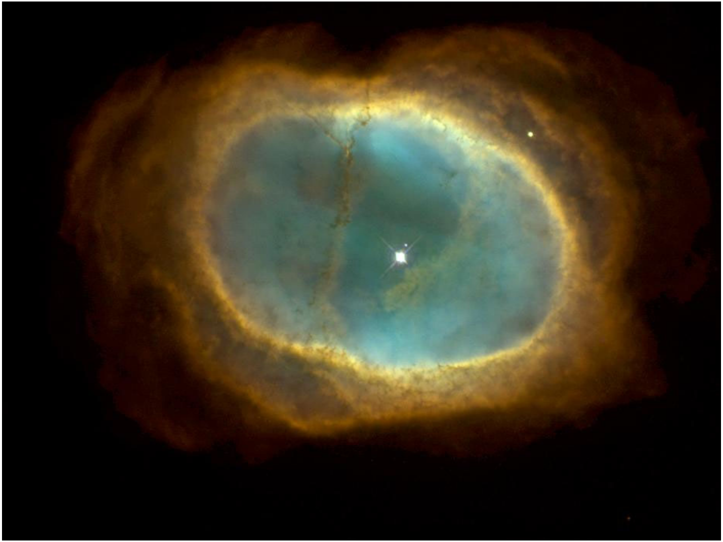
Superfluid Specific Heat

The Specific Heat of Liquid Helium in Zero Gravity very near the Lambda Point from *J. A. Lipa et al* Phys. Rev. **B 68**, 174518 (2003). The specific heat exponent $\alpha = -0.0127 \pm 0.0003$.



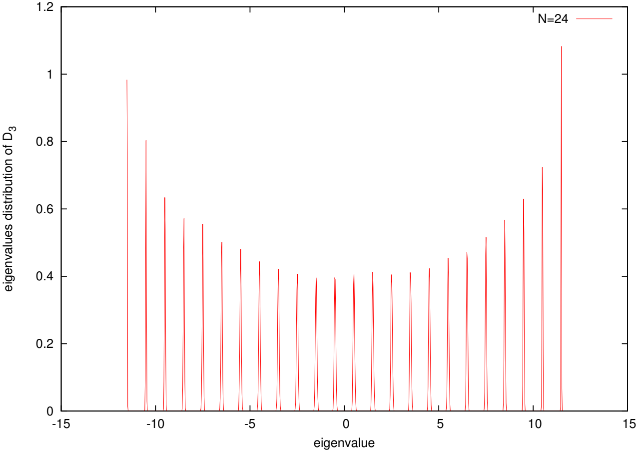
As the temperature is increased the fuzzy sphere expands and evaporates





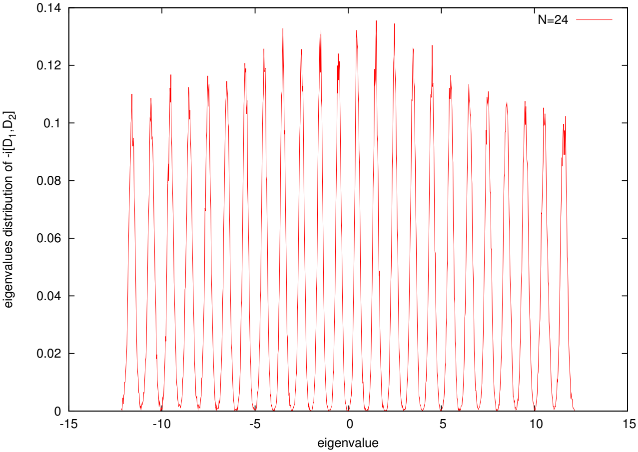
Eigenvalues in the low temperature phase

Eigenvalue distribution of D_3 for $N = 24$.



Eigenvalues in the low temperature phase

Eigenvalue distribution of $[D1, D_2]$ for $N = 24$.

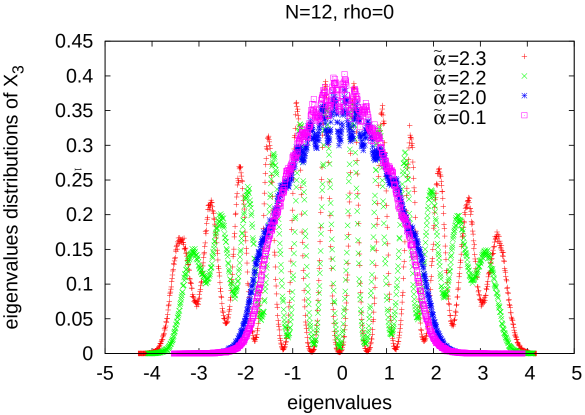


A closer look at the transition

- In the fuzzy sphere phase the eigenvalues fluctuate around the discrete values corresponding to $D_a = L_a$, the irreducible representation of $SU(2)$ of dimension N.
- In the matrix phase, the distribution of eigenvalues of

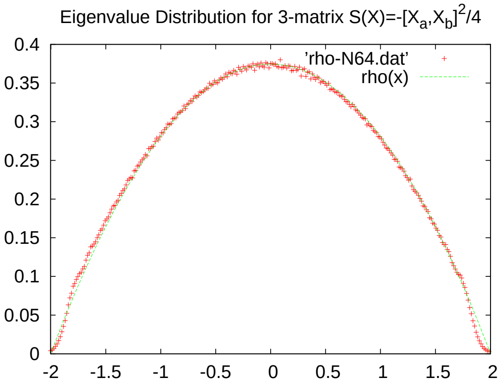
$$X_a = (\frac{\beta}{N^2})^{1/4} D_a = \frac{\tilde{\alpha}}{N^{1/2}} D_a$$

  is largely independent of $\tilde{\alpha}$ and of N.
- In fact fluctuations are around commuting matrices with a uniform distribution in a ball of radius 2. E.g for $N = 12$, the distribution for X_3 ranges from -2 to 2.


N=12, rho=0
eigenvalues distributions of X_3
eigenvalues
$\tilde{\alpha}$=2.3
$\tilde{\alpha}$=2.2
$\tilde{\alpha}$=2.0
$\tilde{\alpha}$=0.1
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
-4
-3
-2
-1
0
1
2
3
4
5

Eigenvalue Spectrum

The spectrum is well fit with $\rho(x) = \frac{3(R^2-x^2)}{4R^3}$ with $R = 2.0$



From solid ball of eigenvalues to fuzzy S^2.

As the system cools a fuzzy S^2 emerges from the ball corresponding the the eigenvalues of the commutating matrices at high temperature.

In passing through the transition the eigenvalue ball of radius 2 expands to a fuzzy sphere of radius $\frac{\sqrt{N}\tilde{\alpha}}{2}$.

Conclusions

- The 3-matrix model provides a concrete model where one can track the geometry as it passes through a phase transition and dissapears.
- The transition is in the geometry. The underlying geometry at a microscopic level is non-commutative— described by a fuzzy sphere with Yang-Mills and matter fluctuations. At high temperatures the eigenvalues form a solid ball of infinitisimal radius on the scale of the fuzzy sphere.
- The geometrical phase emerges as the system cools. This is suggestive of a geometrical phase emerging as the universe cools, or perhaps as the relevant coupling runs to a larger scale.
- Other phase transitions with similar features occur in the dimer model and 6-vertex models.

  It is probable that such transitions belong to a new universality class of topological phase transitions.

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Thank you for your attention!