Gravity as a field theory in flat space-time

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General Relativity is a good classical theory.

But when we construct the quantum gravity big problems arise:

- nonrenormalizability
- formulation of the causality principle
- problem of time choice
- ...

See, for example, review *S. Carlip*, 'Quantum Gravity: a Progress Report', Rept. Prog. Phys, 2001, v. 64, p. 885, arXiv:gr-qc/0108040.

The nature of many problems:

we quantize the gravity following the method which worked well at quantization of field theories in flat space-time,

but if we describe the gravity in terms of variables $g_{\mu\nu}(x)$ then we quantize the structure of the space-time and the geometrical properties of the space-time become operators.

Besides the direct quantization we can search for a workaround:

- string theory
- loop quantum gravity
- ...
- gravity as a field theory in flat space

Regge-Teitelboim embedding theory

Our four-dimensional space-time is a surface in flat space.

The embedding function:

$$y^a(x^\mu) : R^4 \longmapsto R^{1,N-1}, \qquad a = 0, 1, \ldots, N-1. \tag{1}$$

The induced metric:

$$g_{\mu\nu} = \eta_{ab}\,\partial_\mu y^a\,\partial_\nu y^b. \tag{2}$$

The action:

$$S = \int d^4x \sqrt{-g}\, R. \tag{3}$$

Regge-Teitelboim equations:

$$\left(G^{\mu\nu} - \varkappa\, T^{\mu\nu} \right) b^a_{\mu\nu} = 0 \tag{4}$$

(where $b^a_{\mu\nu}$ is the second fundamental form of a surface) include "extra" solutions.

We can either exclude these solutions
or attempt to interpret them.

Regge-Teitelboim embedding theory is a theory of one three-dimensional brane.

This is not a field theory, therefore many problems remain.

The analogy:

mechanics of one particle — embedding theory

continuous medium theory — ???

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A field theory which describes the foliation of flate space $R^{1,N-1}$ into the system of Regge-Teitelboim surfaces.

Assumptions:

- surfaces do not intersect
- surfaces do not interact (maybe, *almost*)

Foliation theory

Let $z^A(y^a)$ be a real field in the flat space $R^{1,N-1}$, $A = 1, \ldots, N-4$.

For each configuration of the field $z^A(y^a)$ there is a foliation of $R^{1,N-1}$ into a system of four-dimensional surfaces $z^A(y) = const$.

We can suppose that one of these surfaces (any of them) is just our space-time.

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Initially there is no coordinates on the surfaces, therefore gravity is described in gauge-invariant terms with respect to general covariance group symmetry.

A new symmetry arises:

$$z^A(y) \longrightarrow z'^A(y) = f^A(z(y)). \tag{5}$$

We can express the projector in terms of $z^A(y)$

$$\begin{aligned} v_a^A &= \partial_a z^A, \qquad & w^{AB} &= v_a^A v_b^B \eta^{ab}, \\ \Pi_{\perp\, ab} &= w_{AB} v_a^A v_b^B, \qquad & \Pi_{ab} &= \eta_{ab} - \Pi_{\perp\, ab}. \end{aligned} \tag{6}$$

In order to work with nonscalar values we must temporarily introduce the coordinates $x^\mu(y)$ on the surfaces.

$\{z^A(y), x^\mu(y)\}$ is a curvilinear coordinates in $R^{1,N-1}$.

Using the embedding formalism we can express all values in terms of $z^A(y)$.
For example, the Riemann curvature tensor is

$$R_{abcd} = \Pi_a^e \Pi_b^f \Pi_c^g \Pi_d^h \left[\left(\partial_e \partial_g z^A \right) w_{AB} \left(\partial_f \partial_h z^B \right) \right]_{gh}, \tag{7}$$

where antisymmetrization operation $[O_{gh}]_{gh} \equiv O_{gh} - O_{hg}$ is used.

Besides $z^A(y)$, the fields of matter can take place in $R^{1,N-1}$.

The action which provides the absence of interaction between the surfaces:

$$S = \int dz\, d^4x \sqrt{-g}\, (R + \mathcal{L}_m)\,. \tag{8}$$

Let only tangent derivatives $\bar{\partial}_d \equiv \Pi_d^e \partial_e$ be used in $\mathcal{L}_m$, for example

$$\mathcal{L}_m = \frac{1}{2} \left(\bar{\partial}_a \varphi\right) \left(\bar{\partial}^a \varphi\right) - V(\varphi) = \frac{1}{2} g^{\mu\nu} \left(\partial_\mu \varphi\right) \left(\partial_\nu \varphi\right) - V(\varphi). \tag{9}$$

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If we change the variables from curvilinear coordinates to rectilinear ones, then we obtain (denoting $w \equiv \det(w^{AB})$)

$$S = \int d^N y \, \sqrt{|w|}\, (R + \mathcal{L}_m) \,. \tag{10}$$

Here the coordinates x^μ on the surfaces are not used.

Equations of motion

The variation of embedding function at arbitrary $\delta z^A(y)$:

$$\delta y^a(x,z) = -\delta z^A \frac{\partial}{\partial z^A} y^a(x,z) - \delta x^\mu \, \partial_\mu y^a(x,z). \qquad (11)$$

The variation of field of matter at arbitrary $\delta z^A(y)$:

$$\delta\varphi(x,z) = \left(\delta z^A \frac{\partial}{\partial z^A} y^a(x,z) + \delta x^\mu \, \partial_\mu y^a(x,z)\right) \partial_a \varphi(y). \qquad (12)$$

The variation of action at arbitrary $\delta z^A(y)$:

$$\delta S = \int dz \, d^4x \left(2\sqrt{-g}\,(G^{\mu\nu} - \varkappa\, T^{\mu\nu})\, b^a_{\mu\nu} \delta y_a(x,z) + \frac{\delta S}{\delta\varphi(x,z)} \delta\varphi(x,z)\right) \qquad (13)$$

The equations of motion for matter:

$$\frac{\delta S}{\delta\varphi(x,z)} = 0 \quad \Leftrightarrow \quad g^{\mu\nu} D_\mu \partial_\nu \varphi + V'(\varphi) = 0. \tag{14}$$

The equations of motion for field $z^A(y)$ coincide with Regge-Teitelboim equations:

$$\left(G^{\mu\nu} - \varkappa\, T^{\mu\nu}\right) b^a_{\mu\nu} = 0. \tag{15}$$

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If we do not use the coordinates x^μ on the surfaces:

$$\begin{gathered} \bar\partial_a \bar\partial^a \varphi + V'(\varphi) = 0, \\ \left(G^{cd} - \varkappa\, T^{cd}\right) b^a_{cd} = 0. \end{gathered} \tag{16}$$

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These equations are invariant with respect the transformation (5), but the action (10) is not invariant.

Possible advantages for quantization

- there is no need to quantize the structure of the space-time
- the gravity is described in gauge-invariant terms, but there is a new symmetry
- the causality principle can be formulated in a usual way
- a 3+1 decomposition arise in a usual way at quantization
- the problem of time choice can possibly be solved
- analysis of divergences and renormalizability is simplified:
    - a dimensional regularization can be used for flat space
    - a new symmetry can lead to some cancellation of divergences

Problems of the approach

- the increase of divergences owing to the increase of space dimensionality
- the problem of extra solutions
    - we can impose additional constraints, but then a preferential direction arise
    - we can try to find boundary conditions at which the solutions do not contradict to observations
- the problem of linearization of equations