Modeling of interaction of QED fields with macroscopical background

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Introduction

In 1948 it was shown by Casimir that vacuum fluctuations of quantum fields generate an attraction between two parallel uncharged conducting planes [H.B.G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948)]

This phenomena called the Casimir effect (CE) has been well investigated with methods of modern experiments

[S.K.Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997), U.Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998); A. Roy, C.-Y. Lin, and U. Mohideen, Phys. Rev. D **60**, 111101(R) (1999), B.W. Harris, F. Chen, and U. Mohideen, Phys. Rev. A **62**, 052109 (2000), G. Bressi et al., Phys. Rev. Lett. **88**, 041804 (2002)].

The CE is a manifestation of influence of fluctuations of quantum fields on the level of classical interaction of material objects.

Theoretical and experimental investigation of phenomena such a kind became very important for development of micro-mechanics and nano-technology.

Though there are many theoretical results on the CE [K.A. Milton, J.Phys. A **37**, R209 (2004)], however the majority of them are received in framework of several models based not on the quantum electrodynamics (QED) directly.

An approach for construction of the single QED model for investigation of all peculiar properties of the CE for thin material films was proposed in [V.N.Markov, Yu.M. Pis'mak, ArXiv:hep-th/0505218, J.Phys. A: Math. Gen. **39**, 6525 (2006) (arXiv:hep-th/0606058), I.V. Fialkovsky, V.N. Markov, Yu.M. Pis'mak, Int. J. Mod. Phys. A **21**,2601 (2006), I.V. Fialkovsky, V.N.Markov, Yu.M. Pis'mak, J.Phys. A: Math. Gen. **41**, 075403 (2008)].

We consider its application for simple forms of films. We show that gauge invariance, locality and renormalizability considered as basic principles make strong restrictions for constructions of the CE models in QED, which make it possible to reveal new important features of the CE-like phenomena.

Formulation of the model

Symanzik action functional (Symanzik K 1981 Nucl. Phys. B 190 1):

$$S(\varphi) = S_V(\varphi) + S_{def}(\varphi)$$

where

$$S_D(\varphi) = \int L(\varphi(x)) d^D x, \ \ S_{def}(\varphi) = \int_\Gamma L_{def}(\varphi(x)) d^{D'} x,$$

and Γ is a subspace of dimension $D' < D$ in D-dimensional space.

From the principles of QED: gauge invariance, locality, renormalizability it follovs that in the model of interaction of material surface with the quantum QED fields the pure photon field contribution can be described with the action functional of the form

$$S(A) = S_0(A) + S_{def}(A)$$

Here $S_0(A)$ - is the usual free action of the photon field $A_\mu(x)$

$$S_0 = -\frac{1}{4} \int d^4 x F^{\mu\nu}(x) F_{\mu\nu}(x),$$
$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x),$$

and $S_{def}(A)$ is the defect action modeling the interaction of field $A_\mu(x)$ with a macroscopic inhomogeneity.

If it is a 2 surface (defect) with the form described by equation $\Phi(x) = 0$, then:

$$S_{def} = \sigma \int d^4x \epsilon^{\nu\mu\lambda\kappa} \partial_\nu \Phi(x) \delta(\Phi(x)) A_\mu(x) \partial_\lambda A_\kappa(x).$$

For the stationary defect $\partial_0 \Phi(x) = 0$ which will be considered the action $S_{def}(A)$ can be written as

$$S_{def}(A) = \sigma \int d^4x \delta(\Phi(x)) \{ 2i A_0(x) \vec{L}_\Phi \vec{A}(x) + \vec{\partial}\Phi[\vec{A}(x) \times \partial_0 \vec{A}(x)] \}$$

where $\vec{L}_\Phi \equiv i[\vec{\partial}\Phi \times \vec{\partial}]$ and σ is a dimensionless coupling constant.

For the sphere with radius r_0 :

$$\Phi(x) = \sqrt{x_1^2 + x_2^2 + x_3^2} - r_0,$$

$$\vec{\partial}\Phi(x) = \frac{\vec{x}}{|\vec{x}|} = \vec{n}(\vec{x}), \quad \vec{L}_\Phi = \frac{1}{|\vec{x}|}i[\vec{x} \times \vec{\partial}] = \frac{1}{|\vec{x}|}\vec{L}$$

The limit $\sigma \to \infty$ corresponds to ideal conducting surface with with conditions $n_\mu \tilde{F}^{\mu\nu}|_S = 0$.

Regularization

To remove the ultraviolet divergencies we use the Pauli-Willars regularization:

$$S_0 \rightarrow S_{0r} = -\frac{1}{4} \int d^4 x F^{\mu\nu}(x)(1 + M^{-2}\partial_\lambda \partial^\lambda)F_{\mu\nu}(x)$$

$$S(A) \rightarrow S_r(A) = S_{0r} + S_{def}$$

and use for calculations the Euclidean version of the action S_E, which is obtained by replations

$$x_0 \rightarrow -ix_0, \quad \partial_0 \rightarrow i\partial_0, \quad A_0 \rightarrow iA_0, \quad a \rightarrow ia.$$

In this case

$$F^{\mu\nu}(x)F_{\mu\nu}(x) \to F_{\mu\nu}(x)F_{\mu\nu}(x), \quad d^4x \to -i d^4x,$$

$$2iA_0(x)\vec{L}_\Phi \vec{A}(x) + \vec{\partial}\Phi[\vec{A}(x) \times \partial_0 \vec{A}(x)] \to$$

$$\to -2A_0(x)\vec{L}_\Phi \vec{A}(x) + i\vec{\partial}\Phi[\vec{A}(x) \times \partial_0 \vec{A}(x)].$$

Thus, $iS_r \to -S_{Er}$, where

$$S_{Er} = \frac{1}{4} \int d^4x \{ M^{-2} F_{\mu\nu}(x)(M^2 - \partial^2) F_{\mu\nu}(x) +$$

$$+ i\sigma\delta(\Phi(x))(2A_0(x)\vec{L}_\Phi \vec{A}(x) - i\vec{\partial}\Phi[\vec{A}(x) \times \partial_0 \vec{A}(x)]) \}.$$

Casimir energy

For the Casimir energy E_{Cas} holds the expression

$$E_{Cas} = -\frac{1}{T} Tr \ln(DD_0^{-1})$$

where D is the propagator in the model with defect, and D_0 is the propagator for the model in homogenous space. For the spherical defect it diverges by $M \to \infty$.

Divergences and renormalization

The asymptotic of the regularized Casimir energy of the spherical defect with raduis r for large M has the form:

$$E_{Cas} = M^3 r_0^2 A(\sigma) + M B(\sigma) + \frac{F(\sigma)}{r_0} + O\left(\frac{1}{M}\right).$$

with

$$F(\sigma) = \frac{3}{64} \frac{\frac{1}{4}\sigma^2}{1 + \frac{1}{4}\sigma^2} + \frac{1}{2\pi} \sum_{l=1}^{+\infty} (2l+1) \times$$

$$\times \int_0^\infty dp \quad \ln \frac{1 - \sigma^2 \mathcal{G}_l(p) \mathcal{R}_l(p)}{1 + \frac{1}{4}\sigma^2} + \frac{\frac{\sigma^2}{4}(2l+1)^4}{\left(1 + \frac{1}{4}\sigma^2\right)\left(4p^2 + (2l+1)^2\right)^3} \Bigg).$$

Here the following notations are used:

$$\mathcal{G}_l(x) = I_{l+\frac{1}{2}}(x) K_{l+\frac{1}{2}}(x),$$

$$\mathcal{R}_l(x) = \left(\frac{1}{2} I_{l+\frac{1}{2}}(x) + I'_{l+\frac{1}{2}}(x) \right) \left(\frac{1}{2} K_{l+\frac{1}{2}}(x) + K'_{l+\frac{1}{2}}(x) \right).$$

with Bessel function $I_{l+\frac{1}{2}}(x)$, $K_{l+\frac{1}{2}}(x)$.

It is finite for finite M but diverges for removing of regularization $M \to \infty$. This problem is solved by the renormalization.

For $\sigma \to \infty$ we opbtaine

$$F_\infty = F(\sigma)|_{\sigma \to \infty} = \frac{3}{64} + \frac{1}{2\pi} \sum_{l=1}^{+\infty} (2l+1) \times$$

$$\times \int_0^\infty dp \left\{ \ln[-4\mathcal{G}_l(p)\mathcal{R}_l(p)] + \frac{(2l+1)^4}{(4p^2 + (2l+1)^2)^3} \right\}.$$

It the results for ideal connecting sphere $E_{Caz} = F_\infty/r_0$, coinciding with one obtained by Boyer.

For removing of the divergences of Casimir energy in the framework of usual multiplicative renormalization procedure one needs to add to the action the terms without photon field with Lagrangian

$$L_{cl}(x) = (Ar_0^2 + B)\delta(|\vec{x}| - r_0),$$

having two constant parameters A, B . Making renormalization of them one can cancel the divergences and obtain the finite renirmalized Casimir energy

$$E_{Cas} = 4\pi r_0^2 \alpha + \beta + \frac{F(\sigma)}{r_0}$$

with finite parameters α, β of dimension of surface energy density and energy. If $\alpha > 0$, $F(\sigma) > 0$ the function E_{Cas} has minimum with $r_0 = \sqrt[3]{F(\sigma)/8\pi\alpha}$.

Casimir-Polder effect

Casimir-Polder effect was predicted theoretically in 1948 (H.B.G.Casimir and D.Polder, Phys.Rev. 73, 360 (1948)). Casimir and Polder found the energy of a neutral point atom in its ground state in the presence of a perfectly conducting infinite plate. In the case of a perfectly conducting plate one can say that the interaction of a fluctuating dipole with the electric field of its image yields the Casimir-Polder potential.

Model

In our model the interaction of the plane surface $x_3 = 0$ with a quantum electromagnetic field A_μ is described by the action:

$$S_{def}(A) = a \int \epsilon^{\alpha\beta\gamma 3} A_\alpha(x) \partial_\beta A_\gamma(x) \delta(x_3) dx.$$

We will use latin indices for the components of 4- tensors with numbers $0, 1, 2$, also the following notations:

$$P^{lm}(\vec{k}) = g^{lm} - k^l k^m / \vec{k}^2,$$

$$L^{lm}(\vec{k}) = \epsilon^{lmn3} k_n / |\vec{k}|, \ \ \vec{k}^2 = k_0^2 - k_1^2 - k_2^2,$$

where $|\vec{k}| = \sqrt{\vec{k}^2}$, and g - metric tensor.

The atom is modeled as a localized electric dipole at the point $(x_1, x_2, x_3) = (0, 0, l)$, which is described by the current $J_\mu(x)$:

$$J_0(x) = \sum_{i=1}^{3} p_i(t)\partial^i \delta(x_1)\delta(x_2)\delta(x_3 - l),$$

$$J_i(x) = -\dot{p}_i(t)\delta(x_1)\delta(x_2)\delta(x_3 - l), \;\; i = 1, 2, 3.$$

The condition of the current conservation holds:

$$\partial_\mu J^\mu = 0,$$

$p_i(t)$ is a function with a zero average and the pair correlator

$$\langle p_j(t_1) p_k(t_2) \rangle = -i \int_{-\infty}^{+\infty} \frac{e^{-i\omega(t_1 - t_2)}}{2\pi} \, \alpha_{jk}(\omega) d\omega,$$

where $\alpha_{jk}(\omega)$ for $\omega > 0$ coincides with the atomic polarizability.

The aim is to calculate the interaction energy E of the atom with a plane, and we will use the following representation for the energy:

$$E = \frac{i}{T} \left\langle \left\{ \ln \int \exp\left(iS(A) + JA\right) DA - \ln \int \exp\left(iS(A)\right) DA \right\}_{(a)} \right\rangle,$$

$\{\cdots\}_{(a)}$ means that the $a = 0$ value of the a-dependent function has to be subtracted: $\{f(a)\}_{(a)} \equiv f(a) - f(0)$.

The ground state energy of a neutral atom in the presence of a plane with Chern-Simons interaction is obtained in the form:

$$E = -\frac{1}{64\pi^2 l^3}\frac{a^2}{1+a^2}\left(\int_0^{+\infty} d\omega e^{-2\omega l}2(1+2\omega l)\alpha_{33}(i\omega)\right.$$

$$\left.+\int_0^{+\infty} d\omega e^{-2\omega l}(1+2\omega l+4\omega^2 l^2)\left(\alpha_{11}(i\omega)+\alpha_{22}(i\omega)\right)\right)$$

$$+\frac{1}{64\pi^2 l^2}\frac{a}{1+a^2}\int_0^{+\infty} d\omega e^{-2\omega l}2\omega\left(1+2\omega l\right)\left(\alpha_{12}(i\omega)-\alpha_{21}(i\omega)\right)$$

It yields the well known Casimir-Polder potential in the limit $a \to +\infty$. The part of the formula with diagonal matrix elements of matrix $\alpha_{jk}(i\omega)$ is equal $a^2/(1 + a^2)$ times the Casimir-Polder interaction of a neutral atom with a perfectly conducting plane. The last line of the formula is odd in a and contains the antisymmetric combination of off-diagonal elements of the atomic polarizability.

It is interesting to analyze the contribution in the energy E from the off-diagonal elements of the atomic polarizability to the potential in more detail. The atomic polarizability can be expressed in terms of dipole matrix elements:

$$\alpha_{jk}(\omega) = \sum_n \left(\frac{\langle 0|d_j|n\rangle \langle n|d_k|0\rangle}{\omega_{n0} - \omega - i\epsilon} + \frac{\langle 0|d_k|n\rangle \langle n|d_j|0\rangle}{\omega_{n0} + \omega - i\epsilon} \right),$$

ω_{n0} is a transition energy between the excited state $|n\rangle$ of the atom and its ground state $|0\rangle$, $\vec{d}$ is a dipole moment operator in the Schrodinger representation. The symmetric $\alpha_{jk}^S(\omega)$ and antisymmetric $\alpha_{jk}^A(\omega)$ parts of $\alpha_{jk}(\omega) = \alpha_{jk}^S(\omega) + \alpha_{jk}^A(\omega)$ can be written as follows:

$$\alpha_{jk}^S(\omega) = \sum_n \frac{2\omega_{n0}\mathrm{Re}M_{jk}^n}{\omega_{n0}^2 - \omega^2} = \alpha_{kj}^S(\omega),$$

$$\alpha_{jk}^A(\omega) = \sum_n \frac{2i\omega\mathrm{Im}M_{jk}^n}{\omega_{n0}^2 - \omega^2} = -\alpha_{kj}^A(\omega),$$

$$M_{jk}^n \equiv \langle 0|d_j|n\rangle\langle n|d_k|0\rangle.$$

Thus, the contribution of $\alpha_{jk}^A(\omega)$ to the potential is different from zero when matrix elements of a dipole moment operator have imaginary parts.

Consider the system with a nonzero $\alpha_{jk}^A(\omega)$ and assume for simplicity the one mode model of the atomic polarizability with a characteristic frequency ω_{10}. Then $\alpha_{12}^A(\omega) = i\omega C_2/(2(\omega_{10}^2 - \omega^2))$, where C_2 is a real constant. In the limit of large separations $\omega_{10}l \gg 1$ we obtain :

$$E|_{\omega_{01}l \gg 1} = -\frac{a^2}{1+a^2} \frac{\alpha_{11}(0) + \alpha_{22}(0) + \alpha_{33}(0)}{32\pi^2 l^4} - \frac{a}{1+a^2} \frac{C_2}{32\pi^2 \omega_{10}^2 l^5}$$

At large enough separations the first term in $E|_{\omega_{01}l \gg 1}$ always dominates. Assuming for simplicity $\alpha_{11}(0) = \alpha_{22}(0) = \alpha_{33}(0) = C_1/(3\,\omega_{10})$, C_1 is a positive constant, one can see from () that if the condition $\frac{|a|C_1}{|C_2|} < 1$ holds then for separations $l \lesssim \frac{|C_2|}{|a|C_1\omega_{10}}$ the term with off-diagonal elements of the atomic polarizability (the second term in $E|_{\omega_{01}l \gg 1}$ dominates.

In the limit of short separations ($b \equiv \omega_{10}l \ll 1$) we obtain:

$$E|_{\omega_{01}l \ll 1} = -\frac{1}{64\pi^2 l^3}\frac{a^2}{1+a^2}\int_0^{+\infty} d\omega\Big(\alpha_{11}(i\omega)+\alpha_{22}(i\omega)+2\alpha_{33}(i\omega)\Big)$$

$$-\frac{C_2}{32\pi^2 l^3}\frac{a}{1+a^2}\left(1-\frac{\pi}{2}b+2b^2-\frac{\pi}{2}b^3+\dots\right) \simeq$$

$$\simeq -\frac{1}{32\pi^2 l^3}\left(\frac{a^2}{1+a^2}C_1\frac{\pi}{3}+\frac{a}{1+a^2}C_2\right) \text{ for } b\to 0.$$

Hence, if the condition $\frac{|a|C_1}{|C_2|}\frac{\pi}{3} < 1$ holds then the term with off-diagonal elements of the atomic polarizability dominates in $E|_{\omega_{01}l \ll 1}$ in the limit of short separations. Thus if we consider the one mode model for the atomic polarizability and the criterion $|a| \lesssim \frac{|C_2|}{C_1}$ holds then the antisymmetric part of the atomic polarizability plays a dominant role in the interaction of the atom with the Chern-Simons plane.

The fermion defect action can be written as

$$S_\Phi(\bar\psi, \psi) = \tag{2}$$

$$= \int \bar\psi(x)[\lambda + u^\mu\gamma_\mu + \gamma_5(\tau + v^\mu\gamma_\mu) + \omega^{\mu\nu}\sigma_{\mu\nu}]\psi(x)\delta(\Phi(x))dx$$

Here, γ_μ, $\mu = 0, 1, 2, 3$, are the Dirac matrices, $\gamma_5 = i\gamma_0\gamma_1\gamma_3\gamma_3$, $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and λ, τ, u_μ, v_μ, $\omega^{\mu\nu} = -\omega^{\nu\mu}$, $\mu, \nu = 0, 1, 2, 3$ are 16 dimensionless parameters.

Expressions (1), (2) are the most general forms of gauge invariant actions concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

We consider CE-like phenomena arising on the distances from the defect boundary much larger then Compton wavelength of the electron.

In this case one can neglect the Dirac fields in QED because of exponential damping of fluctuations of those on much smaller distances ($\sim m_e^{-1} \approx 10^{-10} cm$ for electron, $\sim m_p^{-1} \approx 10^{-13} cm$ for proton [8]).

Thus, for constructing of model we can use the action of free quantum electromagnetic field (photodynamic) with additional defect action (1).

In order to expose essential features of CE-like phenomena in constructed model, we calculate the Casimir force (CF) for simple case of two parallel infinite plane films and study a scattering of electromagnetic wave on the plane defect.

We consider also an interaction of the plane film with a parallel to it straight line current and an interaction of film with a point charge and homogeneous charge distribution on parallel plane.

Casimir force

We consider defect concentrated on two parallel planes $x_3 = 0$ and $x_3 = r$. For this model, it is convenient to use a notation like $x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$.

Defect action (1) has the form:

$$S_{2P} = \frac{1}{2} \int (a_1 \delta(x_3) + a_2 \delta(x_3 - \text{r})) \varepsilon^{3\mu\nu\rho} A_\mu(x) F_{\nu\rho}(x) dx.$$

It is the main point in our model formulation, and no any boundary conditions are used.

The action S_{2P} is translationally invariant with respect to coordinates x_i, $i = 0, 1, 2$. The propagator $D_\Phi(x, y)$ is written as:

$$D_{2P}(x, y) = \frac{1}{(2\pi)^3} \int D_{2P}(\vec{k}, x_3, y_3) e^{i\vec{k}(\vec{x}-\vec{y})} d\vec{k},$$

and $D_{2P}(\vec{k}, x_3, y_3)$ can be calculated exactly.

Using latin indexes for the components of 4-tensors with numbers $0, 1, 2$ and notations

$$P^{lm}(\vec{k}) = g^{lm} - k^l k^m / \vec{k}^2, \ L^{lm}(\vec{k}) = \epsilon^{lmn3} k_n / |\vec{k}|, \ P^{lm} = L_n^l L^{nm},$$

$$\vec{k}^2 = k_0^2 - k_1^2 - k_2^2, \ |\vec{k}| = \sqrt{\vec{k}^2}$$

(g is metrics tensor), one can present the results for the Coulomb-like gauge $\partial_0 A^0 + \partial_1 A^1 + \partial_2 A^2 = 0$ as follows

[V.N. Markov,Yu.M. Pis'mak, ArXiv:hep-th/0505218]

$$D_{2P}^{33}(\vec{k}, x_3, y_3) = \frac{-i\delta(x_3 - y_3)}{|\vec{k}|^2},$$

$$D_{2P}^{l3}(\vec{k}, x_3, y_3) = D_{2P}^{3m}(\vec{k}, x_3, y_3) = 0,$$

$$D_{2P}^{lm}(\vec{k}, x_3, y_3) = \frac{P^{lm}(\vec{k})\mathcal{P}_1(\vec{k}, x_3, y_3) + L^{lm}(\vec{k})\mathcal{P}_2(\vec{k}, x_3, y_3)}{2|\vec{k}|[(1 + a_1 a_2(e^{2i|\vec{k}|\mathrm{r}} - 1))^2 + (a_1 + a_2)^2]}$$

where

$$\mathcal{P}_1(\vec{k}, x_3, y_3) = [a_1 a_2 - a_1^2 a_2^2 (1 - e^{2i|\vec{k}|\mathrm{r}})] \times$$

$$\times [e^{i|\vec{k}|(|x_3|+|y_3-\mathrm{r}|)} + e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3|)}] e^{i|\vec{k}|\mathrm{r}} +$$

$$+ [a_1^2 + a_1^2 a_2^2 (1 - e^{2i|\vec{k}|\mathrm{r}})] e^{i|\vec{k}|(|x_3|+|y_3|)} +$$

$$+ [a_2^2 + a_1^2 a_2^2 (1 - e^{2i|\vec{k}|\mathrm{r}})] e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3-\mathrm{r}|)} -$$

$$- e^{i|\vec{k}||x_3-y_3|} [(1 + a_1 a_2 (e^{2i|\vec{k}|\mathrm{r}} - 1))^2 + (a_1 + a_2)^2],$$

$$\mathcal{P}_2(\vec{k}, x_3, y_3) = a_1 [1 + a_2 (a_2 + a_1 e^{2i|\vec{k}|\mathrm{r}})] e^{i|\vec{k}|(|x_3|+|y_3|)} +$$

$$+ a_2 [1 + a_1 (a_1 + a_2 e^{2i|\vec{k}|\mathrm{r}})] e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3-\mathrm{r}|)} -$$

$$- a_1 a_2 (a_1 + a_2) \left(e^{i|\vec{k}|(|x_3|+|y_3-\mathrm{r}|)} + e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3|)} \right) e^{i|\vec{k}|\mathrm{r}}.$$

The energy density E_{2P} of defect is defined as

$$\ln G(0) = \frac{1}{2} \operatorname{Tr} \ln(D_{2P} D^{-1}) = -iTSE_{2P}$$

where $T = \int dx_0$ is duration of defect, and $S = \int dx_1 dx_2$, is the area of film.

It is expressed in an explicit form in terms of polylogarithm function $\operatorname{Li}_4(x)$ [V.N. Markov,Yu.M. Pis'mak, ArXiv: hep-th/0505218].

For identical films with $a_1 = a_2 = a$ it holds:

$$E_{2P} = 2E_s + E_{Cas}, E_s = \int \ln \sqrt{(1 + a^2)} \frac{d\vec{k}}{(2\pi)^3},$$

$$E_{Cas} = -\frac{1}{16\pi^2 r^3} \left\{ \mathrm{Li}_4 \left(\frac{a^2}{(a+i)^2} \right) + \mathrm{Li}_4 \left(\frac{a^2}{(a-i)^2} \right) \right\}.$$

Here E_s is an infinite constant, which can be interpreted as self-energy density on the plane, and E_{Cas} is an energy density of their interaction.

The function $\mathrm{Li}_4(x)$ is defined as

$$\mathrm{Li}_4(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^4} = -\frac{1}{2} \int_0^{\infty} k^2 \ln(1 - xe^{-k})dk.$$

The force $F_{2P}(\mathrm{r}, a)$ between planes is given by

$$F_{2P}(\mathrm{r}, a) = -\frac{\partial E_{Cas}(\mathrm{r}, a)}{\partial \mathrm{r}} = -\frac{\pi^2}{240\mathrm{r}^4}f(a).$$
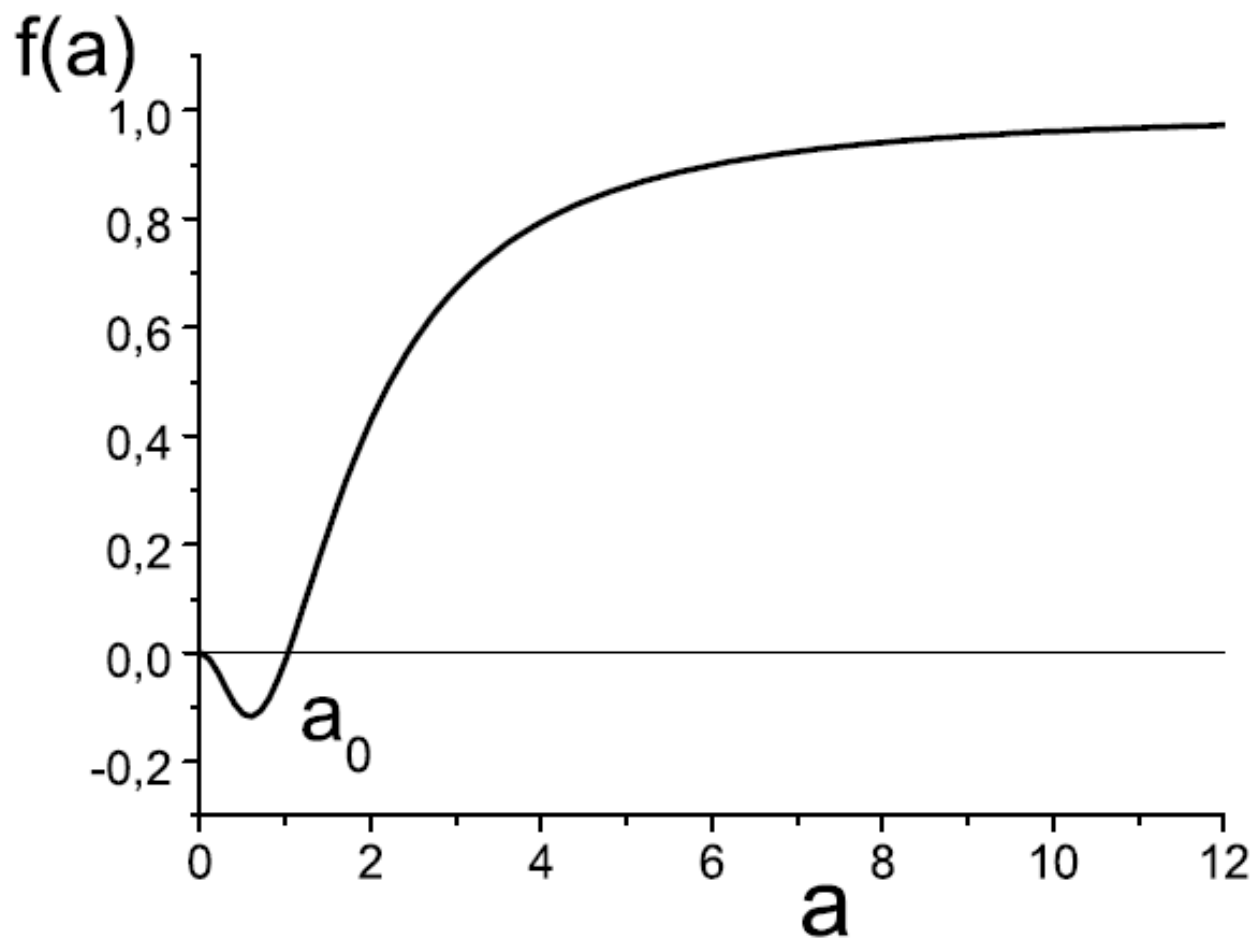
Figure 1: Function $f(a)$ determining Casimir force between parallel planes

The force F_{2P} is repulsive for $|a| < a_0$ and attractive for $|a| > a_0$, $a_0 \approx 1.03246$ (see Figure 1).

For large $|a|$ it is the same as the usual CF between perfectly conducting planes. The model predicts that the maximal magnitude of the *repulsive* F_{2P} is expected for $|a| \approx 0.6$.

For two infinitely thick parallel slabs the repulsive CF was predicted also in [O. Kenneth et al., Phys. Rev. Lett. **89**, 033001 (2002)].

Real film has a finite width, and the bulk contributions to the CF for nonperfectly conducting slabs with widths h_1, h_2 are proportional to $h_1 h_2$. Therefore it follows directly from the dimensional analysis that the bulk correction F_{bulk} to the CF is of the form $F_{bulk} \approx c F_{Cas} h_1 h_2 / r^2$ where F_{Cas} is the CF for perfectly conducting planes and c is a dimensionless constant. This estimation can be relevant for modern experiments on the CE.

For instance, in [G. Bressi et al., Phys. Rev. Lett. **88**, 041804 (2002)] there were results obtained for parallel metallic surfaces where width of layer was about $h \approx 50$ nm and typical distance r between surfaces was $0.5\mu m \leq r \leq 3\mu m$. In that case $3 \times 10^{-4} \leq (h/r)^2 \leq 10^{-2}$.

In [G. Bressi et al., Phys. Rev. Lett. **88**, 041804 (2002)] the authors have fitted the CF between chromium films with function C_{Cas}/r^4. They claim that the value of C_{Cas} coincides with known Casimir result within a 15% accuracy.

It means that bulk force can be neglected, and only surface effects are essential. In our model the values $a > 4.8$ of defect coupling parameter a are in good agreement with results of [G. Bressi et al., Phys. Rev. Lett. **88**, 041804 (2002)].

Interaction of film with classical current and electromagnetic waves

Now we study the scattering of classical electromagnetic wave on plane defect and effects generated by coupling of plane film with a given classical 4-current.

The scattering problem is described in our approach by a homogeneous classical equation $K_{2P}^{\mu\nu}A_\nu = 0$ of simplified model with $a_1 = a$, $a_2 = 0$. It has a solution in the form of a plane wave.

If one defines transmission (reflection) coefficient as a ratio $K_t = U_t/U_{in}$, $(K_r = U_r/U_{in})$ of transmitted wave energy U_t (reflected wave energy U_r) to incident wave energy U_{in}, then direct calculations give the following result:

$$K_t = (1 + a^2)^{-1}, K_r = a^2(1 + a^2)^{-1}$$

We note the following features of reflection and transmission coefficients. In the limit of infinitely large defect coupling these coefficients coincide with coefficients for a perfectly conducting plane.

The reflection and transmission coefficients do not depend on the incidence angle.

The classical charge and the wire with current near defect plane are modeled by appropriately chosen 4-current J in (3).

The mean vector potential $\mathcal{A}_\mu$ generated by J and the plane $x_3 = 0$, with $a_1 = a$ can be calculated as

$$\mathcal{A}^\mu = -i \frac{\delta G(J)}{\delta J_\mu}\Big|_{a_1=a, a_2=0} = i D_{2P}^{\mu\nu} J_\nu|_{a_1=a, a_2=0}. \qquad (5)$$

Using notations $\mathcal{F}_{ik} = \partial_i \mathcal{A}_k - \partial_k \mathcal{A}_i$, one can present electric and magnetic fields as $\vec{E} = (\mathcal{F}_{01}, \mathcal{F}_{02}, \mathcal{F}_{03})$, $\vec{H} = (\mathcal{F}_{23}, \mathcal{F}_{31}, \mathcal{F}_{12})$.

For charge e at the point $(x_1, x_2, x_3) = (0, 0, l)$, $l > 0$ the corresponding classical 4-current is

$$J_\mu(x) = 4\pi e \delta(x_1) \delta(x_2) \delta(x_3 - l) \delta_{0\mu}$$

In virtue of (5) the mean vector potential $\mathcal{A}^\mu(x)$ is independent on x_0 and the electric field in considered system is defined by potential

$$\mathcal{A}_0(x_1, x_2, x_3) = \frac{e}{\rho_-} - \frac{a^2}{a^2 + 1} \frac{e}{\rho_+}.$$

where $\rho_+ \equiv \sqrt{x_1^2 + x_2^2 + (|x_3| + l)^2}$, $\rho_- \equiv \sqrt{x_1^2 + x_2^2 + (x_3 - l)^2}$.

The electric field $\vec{E} = (E_1, E_2, E_3)$ is of the form

$$E_1 = \frac{ex_1}{\rho_-^3} - \frac{a^2}{a^2+1}\frac{ex_1}{\rho_+^3}, \quad E_2 = \frac{ex_2}{\rho_-^3} - \frac{a^2}{a^2+1}\frac{ex_2}{\rho_+^3},$$

$$E_3 = \frac{e(x_3 - l)}{\rho_-^3} - \frac{a^2\epsilon(x_3)}{a^2+1}\frac{e(|x_3| + l)}{\rho_+^3}.$$

Here, $\epsilon(x_3) \equiv x_3/|x_3|$.

We see that for $x_3 > 0$ the field $\vec{E}$ coincides with field generated in usual classical electrostatic by charge e placed on distance l from infinitely thick slab with dielectric constant $\epsilon = 2a^2 + 1$.

Because $\mathcal{A}^\mu(x) \neq 0$ for $\mu = 1, 2, 3$, the defect generate also a magnetic field $\vec{H} = (H_1, H_2, H_3)$:

$$H_1 = \frac{eax_1}{(a^2 + 1)\rho_+^3}, \quad H_2 = \frac{eax_2}{(a^2 + 1)\rho_+^3}, \quad H_3 = \frac{ea(|x_3| + l)}{(a^2 + 1)\rho_+^3}.$$

It is an anomalous field which doesn't arise in classical electrostatics. Its direction depends on sign of a.

In similar one can calculate the fields generated by interaction of the film and charged plane $x_3 = l$, presented by the classical current

$$J_\mu(x) = 4\pi\sigma\delta(x_3 - l)\delta_{0\mu}.$$

Here σ is the charge density. In this case it holds:

$$E_1 = E_2 = 0 = H_1 = H_2 = 0,$$

$$E_3 = 2\pi\sigma\left(\epsilon(x_3 - l) - \epsilon(x_3)\frac{a^2}{a^2 + 1}\right), \quad H_3 = 2\pi\sigma\frac{a}{a^2 + 1}.$$

Thus, in considered system there is only one dependent on l component of fields $\vec{E}$, $\vec{H}$. It is E_3. For $l \to \mp\infty$

$$E_3 = 2\pi\sigma \left(\pm 1 - \frac{\epsilon(x_3)a^2}{a^2+1} \right),$$

and for $l = 0$

$$E_3 = \frac{2\pi\sigma\epsilon(x_3)}{a^2+1}.$$

It is important, to note that anomalous fields arise because the space parity is broken by the action (4), and they are generated in (5) by the $L\mathcal{P}_2$- term of propagator D_{2P}.

A current with density j flowing in the wire along the x_1-axis is modeled by

$$J_\mu(x) = 4\pi j \delta(x_3 - l)\delta(x_2)\delta_{\mu 1}$$

For magnetic field from (5) one obtains in region $x_3 > 0$ the usual results of classical electrodynamics for the current parallel to infinitely thick slab with permeability $\mu = (2a^2 + 1)^{-1}$. There is also an anomalous electric field $\vec{E} = (0, E_2, E_3)$:

$$E_2 = \frac{2ja}{a^2 + 1}\frac{x_2}{\tau^2}, \quad E_3 = \frac{2ja}{a^2 + 1}\frac{|x_3| + l}{\tau^2}$$

where $\tau = (x_2^2 + (|x_3| + l)^2)^{\frac{1}{2}}$.

Comparing formulae $\epsilon = 2a^2 + 1$ and $\mu = (2a^2 + 1)^{-1}$ for parameter a we obtain the relation $\epsilon\,\mu = 1$.

It holds for material of thick slab interaction of which with point charge and current in classical electrodynamics was compared with results for thin film of our model.

The speed of light in this hypothetical material is equal to one in the vacuum. From the physical point of view, it could be expected, because interaction of film with photon field is a surface effect which can not generate the bulk phenomena like decreasing the speed of light in the considered slab.

The essential property of interaction of films with classical charge and current is the appearance of anomalous fields. This fields are suppressed in respect of usual ones by factor a^{-1} and they vanish in case of perfectly conducting plane.

Magnetoelectric (ME) films are good candidates to detect anomalous fields and non ideal CE. The generic example of ME crystals is Cr_2O_3 [16]. It is important to note that for ME films the Lifshitz theory of CE is not relevant but they can be studied in our approach.

Non-planar geometry

The case of cylindrical film.

$$S_{def} = a/2 \int d^4x \, \varepsilon_{\mu\nu\rho\sigma} \partial^\mu \Phi(x) A^\nu \partial^\rho A^\sigma \delta(\Phi)$$

where A — EM vector-potential, $\varepsilon_{\mu\nu\rho\sigma}$ — totaly antisymmetric tensor $(\varepsilon_{0123} = 1)$, and the defect is described with equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$.

For cylindrical shell placed along the x_3 axis, $x_1^2 + x_2^2 = R^2$ we have
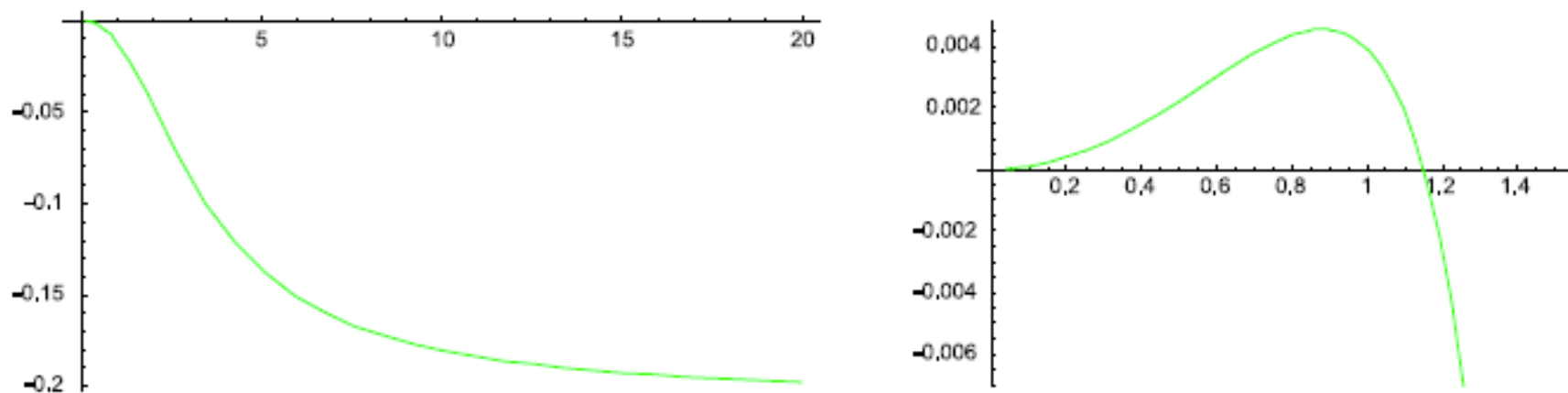
$$\Phi(x) = x_1^2 + x_2^2 - R^2$$

Figure 2: E_{cas} for real a (left) and imaginary a (right).

The result for CE has the form

$$E = E_{div} + E_{fin}$$

$$E_{div} = M^3 R f_3(a) + \frac{M}{R} f_1(a),$$

$$E_{fin} = \frac{E_{cas}(a)}{4\pi R^2} + O(1/M)$$

[I.V. Fialkovsky, V.N.Markov, Yu.M. Pis'mak, J.Phys. A: Math. Gen. **41**, 075403 (2008)]

Electromagnetic fields generated by simplest fermion defects

We consider the fermionic defect of the form

$$S_{\lambda q}(\bar{\psi}, \psi) \equiv \int \bar{\psi}(\vec{x}, 0)(\lambda + \hat{q})\psi(\vec{x}, 0)d\vec{x}$$

Here $\bar{\psi}$, ψ are the Dirac spinor fields, λ is a constant parameter, q is a fixed 4-vectors, $\hat{q} = q_\mu \gamma^\mu$ (γ^μ are the Dirac gamma-matrices), and we used the short hand notation for the 4-vector: $x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$.

Vector $q = (\vec{q}, 0)$ and scalar λ describe the interaction of current and density of Dirac field with material defect. Namely, the zero component of vector $\vec{q}$ defines a surface charge density and space like components of vector $\vec{q}$ parallel to the defect plane describe the surface current.

The scalar λ defect can be interpreted as a surface mass term.

The interaction of vacuum fluctuations of the Dirac field with the background generates quantum corrections to usual classical effects.

Asymptotics of the generated by the defect electromagnetic fields for large and small x_3 are the following

[I.V. Fialkovsky,V.N. Markov,Yu.M. Pis'mak, Int. J. Mod. Phys. A **21**,2601 (2006)].

If $q = (\kappa, 0, 0, 0) = q^{(1)}$, the defect generates pure electric field E_3

$$E_3 \underset{x_3 \to 0}{\approx} \frac{em^2}{8\pi^2\omega^2} \left[(1 + \omega^2)\mathrm{Arctg}(\omega) - \omega\right] \left(\frac{1}{m^2 x_3^2} - 2\right).$$

$$E_3 \underset{x_3 \to \infty}{\approx} -\frac{em^2}{4\pi^2\omega^2} \left[\mathrm{Arctg}(\omega) - \omega\right], \quad \omega = \frac{4\kappa}{4 - \kappa^2}.$$

For $q = (0, \kappa, 0, 0) \equiv q^{(2)}$ the field is pure magnetic

$$H_2 \underset{x_3 \to 0}{\approx} -\frac{em^2}{16\pi^2\omega'^2}\left[(1 + \omega'^2)\ln\frac{1 + \omega'}{1 - \omega'} - 2\omega'\right]\left(\frac{1}{m^2x_3^2} - 2\right).$$

$$H_2 \underset{x_3 \to \infty}{\approx} \frac{em^2}{8\pi^2\omega^{2'}}\left[\ln\frac{1 + \omega'}{1 - \omega'} - 2\omega'\right], \ \omega' = \frac{4\kappa}{4 + \kappa^2}.$$

For $q = (\kappa, \kappa, 0, 0) \equiv q^{(3)}$, $E_1 = E_2 = H_1 = H_3 = 0$, and asymptotics of the fields E_3, H_2 are of the form

$$E_3 \underset{x_3 \to 0}{\approx} H_2 \underset{x_3 \to 0}{\approx} -\frac{e\kappa m^2}{12\pi^2} \left(\frac{1}{m^2 x_3^2} - 2 \right),$$

$$E_3 \underset{x_3 \to \infty}{\approx} H_2 \underset{x_3 \to \infty}{\approx} -\frac{e\kappa m^2}{12\pi^2}.$$

Conclusion

The main results of our study on the CE for thin films in the QED are the following.

We have shown that if the CF holds true for thin material film, then an interaction of this film with the QED fields can be modeled by photodynamic with the defect action (1) obtained by most general assumptions consistent with locality, gauge invariance and renormalizability of model.

Thus, the basic principles of QED were essential in our studies of the CE. These principles make it possible to expose new peculiarities of the physics of macroscopic objects in QED and must be taken into account for construction of the models.

We calculated the Casimir energy of spherical films interacting with quantum electromagnetic field. The result obtained in the framework of multiplicative renormalization procedure depends on the 3 constant parameters. One of them is dimensionless and is a coupling constant of the sphere with photon field. If it is given, the $1/r_0$-contribution to Casimir energy is calculated exactly. The renormalization procedure forecasts the necessarily existence of the radius independent and the proportional to r_0^2 contributions to Casimit energy described by two parameters.

Thus, the Casimir energy appears to be non-universal and dependent on the properties of material.

The presented approach can by applied for the problem of stability of fullerenes and the other objects of nano-physics.

In the framework of quantum electrodynamics we consider a model with the Chern-Simons action on a two-dimensional plane having one dimensionless parameter a which describes properties of the material. The formula for the energy of interaction of a neutral atom (molecule) with fluctuations of vacuum of the photon field in the presence of a two-dimensional plane with Chern-Simons interaction is derived. In the limiting case $a \to +\infty$ the result coincides with the Casimir-Polder result for the energy of interaction of a neutral atom with a perfectly conducting plane. The essential feature of the result is the term depending on the antisymmetric part of a dipole correlation function for finite values of the parameter a, we derive a criterion of its dominance in terms of imaginary and real parts of dipole matrix elements of the atom and the parameter a of the Chern-Simons surface term.

We expect quantum Hall effect systems and graphene to be the most promising known materials for the measurements of the potential derived in this paper. The Casimir-Polder effect provides a recipe for direct measurements of the parameter a in such materials, which can be relevant for better understanding of quantum dynamics in these systems. The measurements of the antisymmetric part of the atomic polarizability by means of the Casimir-Polder effect can be an independent possibility for study of antisymmetric parts of atomic polarizabilities in various atomic and molecular systems.