

Surprises of Dimensional Reduction

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Reduction of Dimensions

Dimensional reduction, the transition $4D \rightarrow 2D$, was used in 90s in HE Regge scattering (Aref'eva, Lipatov). In XXI, it got impetus in quantum gravity opening the way to (super)renormalizability.

We study the coupling behavior in $g\varphi^4$ model defined in both the 4D, 2D domains; the $\bar{g}(Q^2)$ evolutions being duly conjugated at a reduction scale $Q \sim M$.

[Sh., arXiv: hep-th 1004.1510]

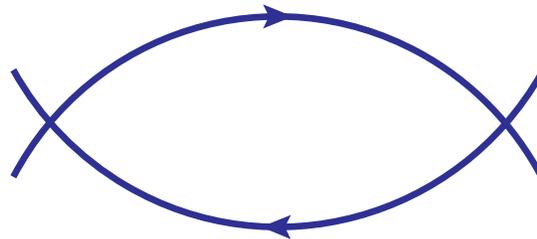
The $g \varphi^4$ model in 4D and 2D

Consider

$$L = T - V; \quad V(m, g; \varphi) = \frac{m^2}{2} \varphi^2 + \frac{4\pi^{d/2} M^{d-4}}{9} g \varphi^4; \quad g > 0$$

in parallel in 4D (space $d=3$) and 2D ($d=1$).

Limit ourselves to 1-loop leading level for \bar{g} corresponding to only diagram, the 1st correction to 4-vertex function,.



Its contribution I enters into running coupling as follows:

$$\bar{g}(q^2) = \frac{g_i}{1 - g_i I(q^2; m^2, m_i^2)}.$$

Smooth DR in the mom picture

DR in Feynman integral by modifying metric

$$dk = d^4 k \rightarrow d_M k = \frac{d^4 k}{1 + k^2/M^2}; \quad k^2 = \mathbf{k}^2 - k_0^2.$$

$$I\left(\frac{q^2}{m^2}\right) = \frac{i}{\pi^2} \int \frac{dk}{(m^2 + k^2)[m^2 + (k + q)^2]} \rightarrow$$
$$\rightarrow \frac{i M^2}{\pi^2} \int \frac{dk}{(m^2 + k^2)[m^2 + (k + q)^2][M^2 + k^2]} = J(\kappa; \mu),$$

with $\kappa = q^2/4m^2$, $\mu = M^2/m^2$, $q^2 = \mathbf{q}^2 - q_0^2$. Explicitely;

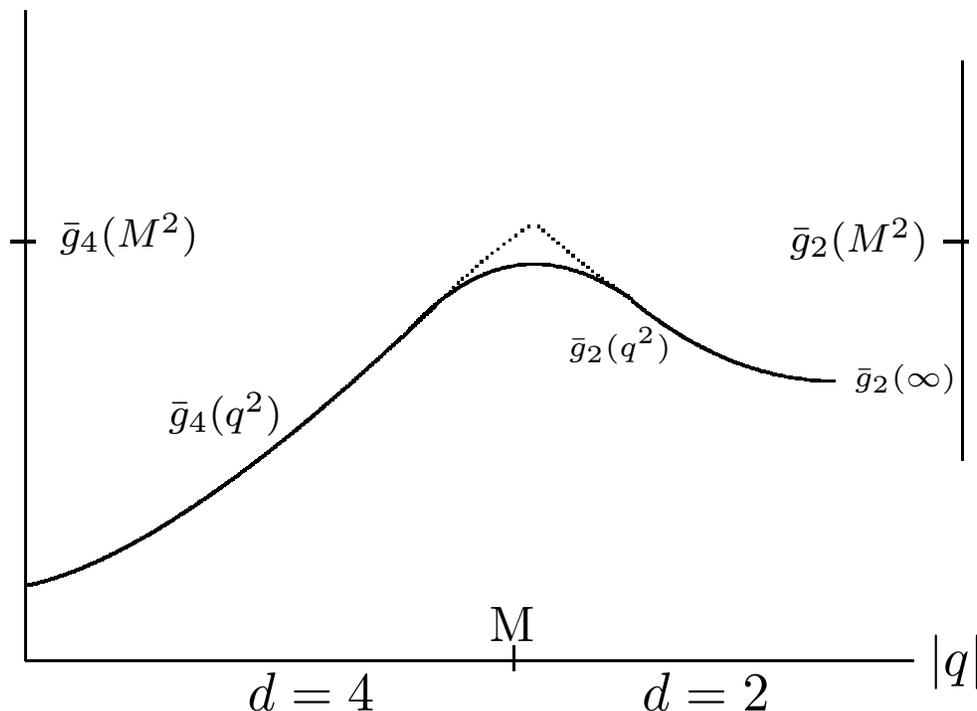
$$J_i^{[4]}(\kappa; \mu) \sim \ln\left(\frac{q^2}{m_i^2}\right); \quad J_i^{[2]}(\kappa; \mu) \sim \ln\left(\frac{4M^2}{m_i^2}\right) + \frac{M^2}{q^2} \ln \frac{q^2}{M^2}$$

at $m^2 \ll q^2 \ll M^2$ and at $M^2 \ll q^2 \sim q^2 \gg M^2$. 1st intermediate asymptote is rising; 2nd, final - **decreasing**.

The \bar{g} UV fixed point by DR

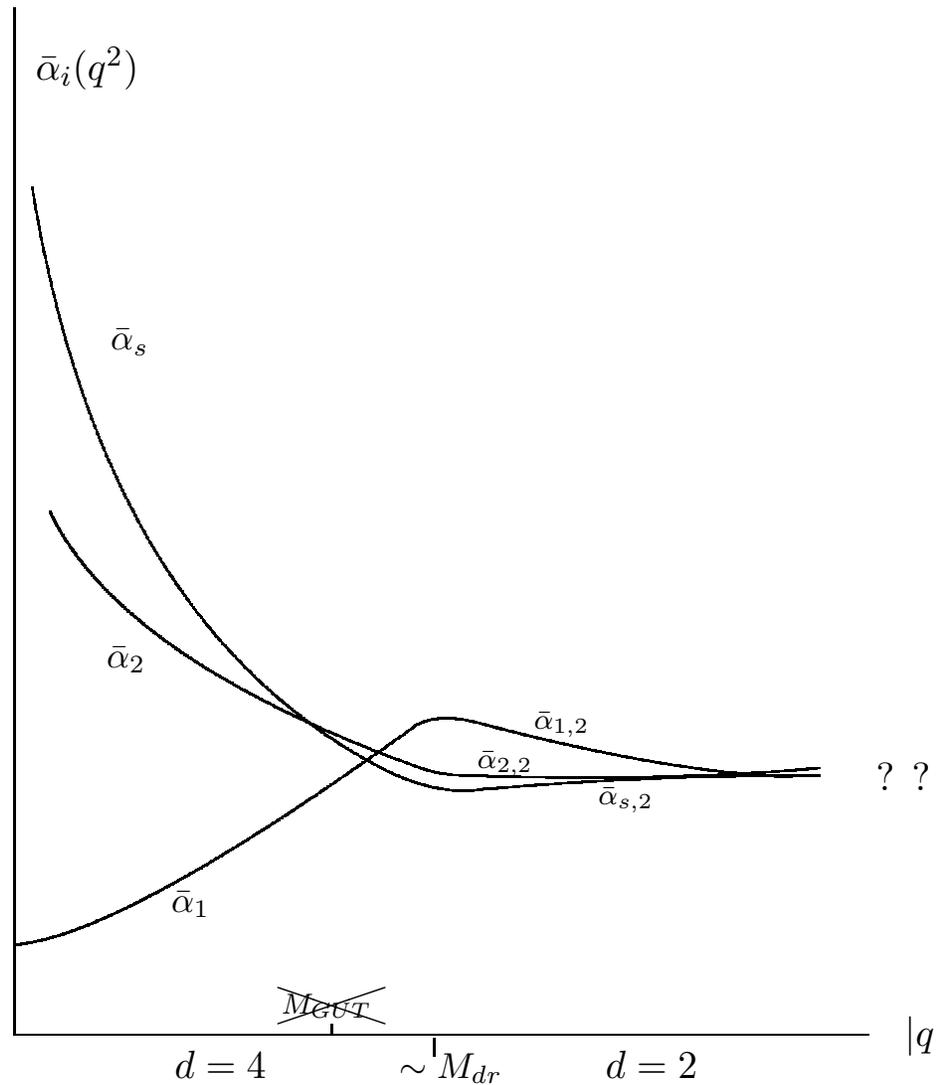
Coupling evolution changes drastically. The $\bar{g}(q^2)$ diminishes beyond DR scale tending to finite value

$$\bar{g}_2(\infty) = \frac{g_M}{1 + g_M I_2(M^2/m^2)} < g_M :$$



The dotted lines correspond to hard conjunction at DR scale.

Great Unification via DR Looking-Glass ?



New brave **Great Unification by DR** instead of leptoquarks.

Resume of the DR hypothesis

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In micro -

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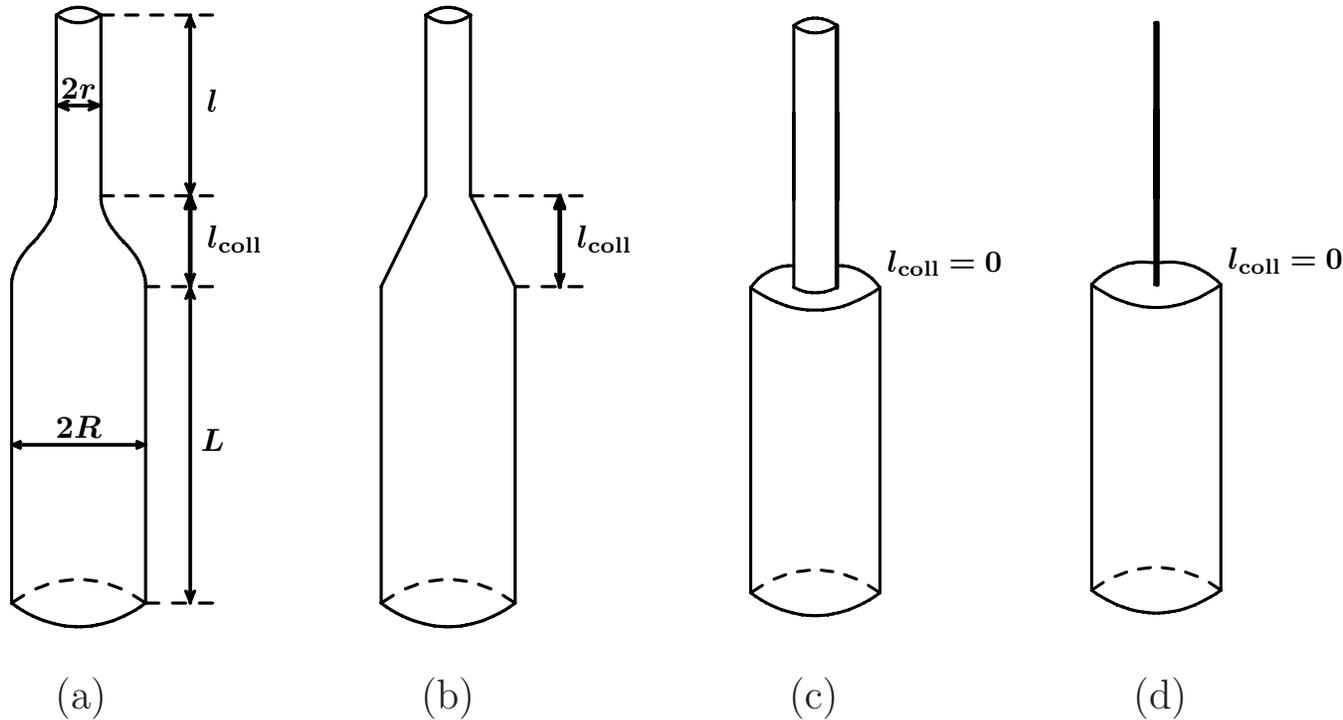
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In micro - and/or in macro-world ?

To this goal, we'll study few toy models ...

Toy models for the DR Looking-Glass

for studying (classic/quantum) problems on variable geometry manifold, like **2-surface** of “bottle” :



– to learn on possible physical signal
“from/through looking-glass at scale M_{DR} ”

Klein-Gordon on toy model geometry

Our attitude does not imply any modification of the concept of time. Instead, we have in mind some continuous reduction of spatial dimensions.

Few toy models of space with variable geometry used. To get physical intuition and experience, we start with Klein-Gordon scalar waves on the bottle surface.

For detail, see [P.Fiziev, D.Sh., arXiv:hep-th 1009.5309] also [Sh.D., Fiziev P., *"Amusing properties of Klein-Gordon solutions ..."* talk at Bogoliubov Readings 2010 ;<http://tcpa.uni-sofia.bg/index.php?n=7>]

large Klein-Gordon on composite space

The Klein-Gordon=KG Eq

$$\square\varphi - M^2\varphi = 0; \quad \square = -\frac{1}{\sqrt{|g|}}\partial_\mu \left(\sqrt{|g|}g^{\mu\nu}\partial_\nu \right) = -\partial_{tt}^2 + \Delta_d, .$$

with d -dim operator Δ_d on the d -dim metric γ_{mn}

$$\Delta_d = \frac{1}{\sqrt{|\gamma|}}\partial_m \left(\sqrt{|\gamma|}\gamma^{mn}\partial_n \right), \quad m, n = 1, \dots, d.$$

At the $(1 + d)$ -formalism with common time
any global solution φ on the parts with different
dimension d_1, d_2, \dots have a common frequency

$$\omega = \omega_1 = \omega_2 = \dots .$$

Klein-Gordon on Cylindrical 2D Space

$$\text{KGEq} \quad \square\varphi - M^2\varphi = -\partial_{tt}^2\varphi + \Delta_2\varphi - M^2\varphi = 0;$$

on 2-dim cylinder-symmetry space with shape function $\rho(z)$

$$(dl)^2 = \gamma_{mn}dx^m dx^n = \rho^2(z) d\phi^2 + (1 + \rho'^2) dz^2,$$

and

$$\Delta_2 = \frac{1}{\rho^2} \left(\partial_{\phi\phi}^2 + \frac{\rho}{\sqrt{1+\rho'^2}} \partial_z \frac{\rho}{\sqrt{1+\rho'^2}} \partial_z \right).$$

Separating variables $\varphi(t, \phi, z) = T(t)\Phi(\phi)Z(z)$ one gets
 $T'' + \omega^2 T(t) = 0$; $\Phi'' + m^2 \Phi(\phi) = 0$; and

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$$\frac{1}{\rho\sqrt{1+\rho'^2}} \partial_z \left(\frac{\rho}{\sqrt{1+\rho'^2}} \partial_z Z \right) + \left(\omega^2 - M^2 - \frac{m^2}{\rho(z)^2} \right) Z(z) = 0, \quad (Z)$$

Centrifugal potential due to motion in curved space.

with orbital number $m \neq 0$

“Dynamics instead of Geometry”

Eq.(Z) can be transformed to a proper Schrödinger-like eq. :

$$\psi''(u) + (E - V(u)) \psi(u) = 0$$

by change of coordinate

$$z \mapsto u : \quad u(z) = \int \sqrt{1 + \rho'(z)^2} \frac{dz}{\rho(z)}$$

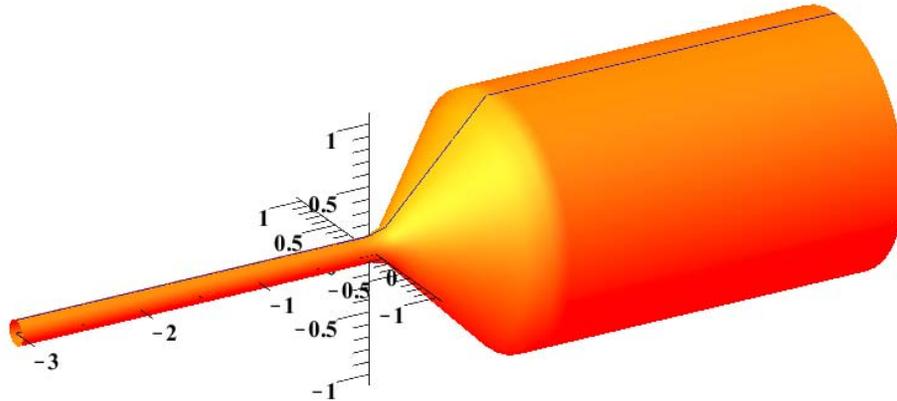
and shape function $\rho(z)$ to $\varrho(u) = \rho(z(u))$. Then

$\Delta_2 = \varrho^{-2} \left(\partial_{\phi\phi}^2 + \partial_{uu}^2 \right)$ [$\varrho(u)^2$ being 2-dim conformal factor],

$$E = -m^2, \quad V(u) = (M^2 - \omega^2) \varrho(u)^2, \quad Z(z) = \psi(u(z)).$$

STATEMENT : Study of KGEq on a curved manifold is reducible to the Schrödinger-like eq solving (with potential $V(u)$ defined by geometry).

KGEq on 2 Cylinders Connected by Part of Cone



The **Surface** of 2 cylinders connected by part of cone with shape function

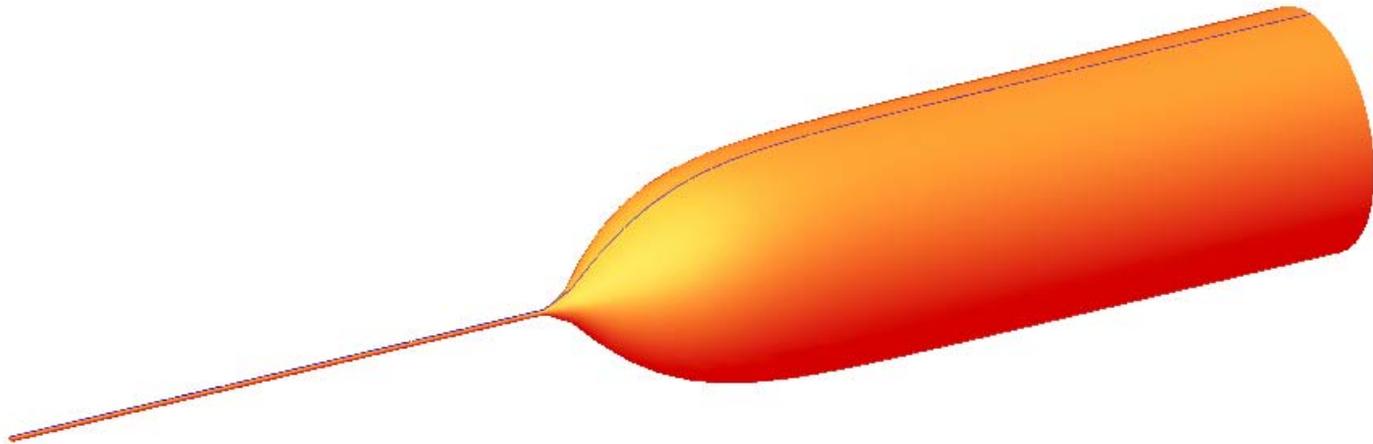
$$\rho_{cone}(z) = \begin{cases} R = \text{const} & : \text{for } z \in [z_R, +\infty), \\ z \tan \alpha & : \text{for } z \in [z_r, z_R], \\ r = \text{const} & : \text{for } z \in (-\infty, z_r]. \end{cases}$$

This model in the (singular) limit $r \rightarrow 0$ describes

Dimension Reduction $d = 2 \rightarrow d = 1$.

Conic Junction with Smooth Transition to R-Cylinder

The ST modification

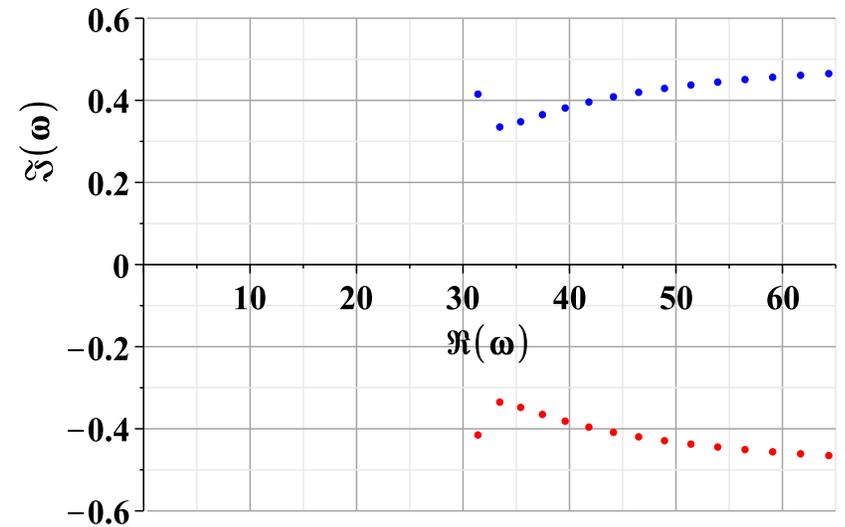
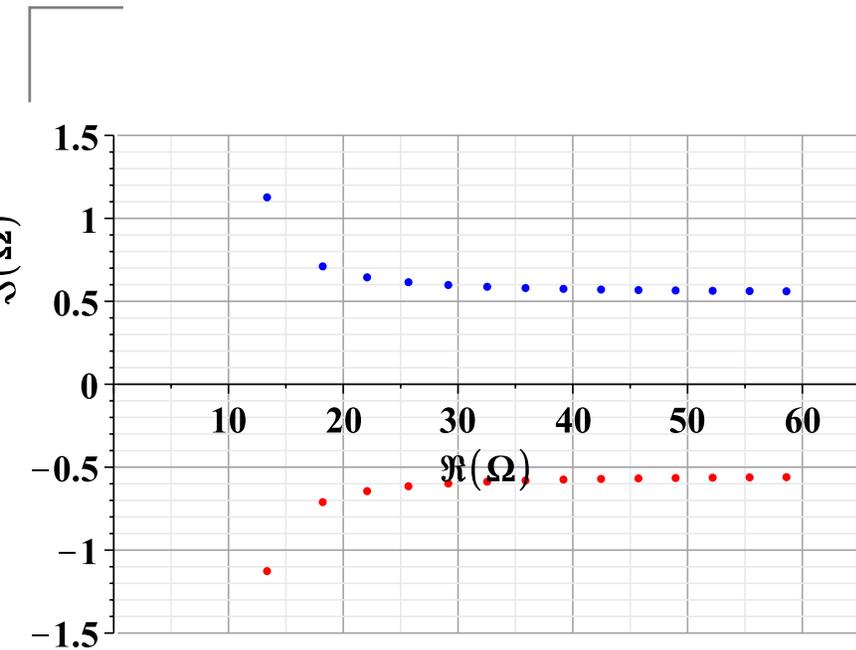


allows one, to find explicit analytical solutions

- via Bessel functions - for class of shapes $\rho_{ST}(z)$ with Smooth Transition to R-Cylinder, define S-matrix and get spectra of KG problem.

Including limiting case $r = 0$ of dim-reduction.

Spectra for “2 Cylinders + Part of Cone surface”

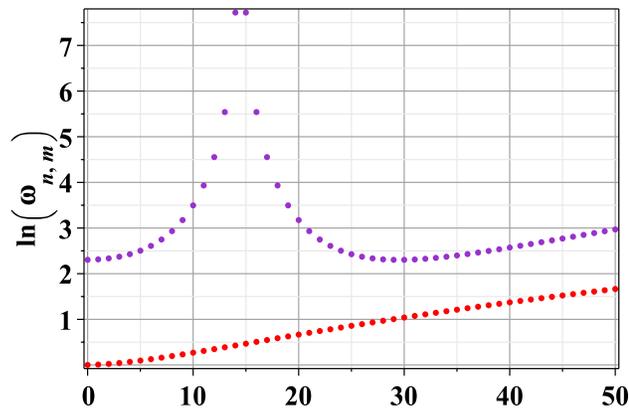


Complex spectra $\omega_{n,m}^Q = \Re(\Omega) + i \Im(\Omega)$, $n = 0, 1, 2, \dots$ for $m = 10$, $\alpha = \pi/3$ and masses $M = 0$ (left), $M = 28.85$ (right) vs. dimensionless variable $\Omega = R \sqrt{\omega^2 - M^2} / \sin \alpha$.

Spectra for “2 Cylinders + smooth junction”= ST

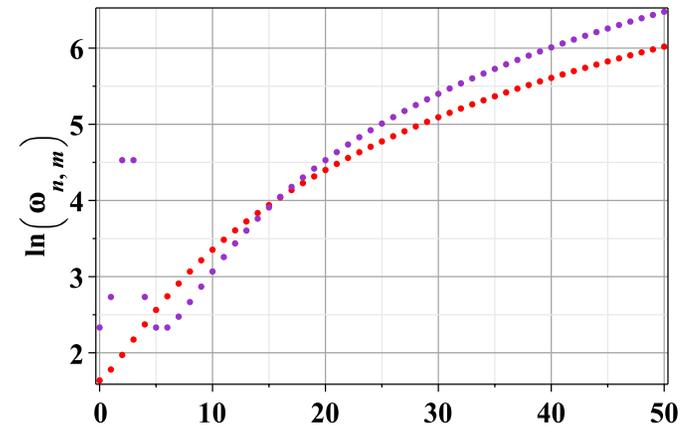
In the ρ_{ST} case there are 2 series of real frequencies.

The related spectra depend on the cone angle α :



$$\sin \alpha = 0.2$$

Red = $\omega_{n,+}$, Blue = $\omega_{n,-}$



$$\sin \alpha = 1$$

Comparing spectra of ρ_{cone} case with ρ_{ST} ones suggests an idea:

To generate and explain observed spectra of particles using proper geometry of the junction between domains of space with different dimensions.

Studying Spectra for Variable Geometry

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Thus, spectrum is a specific "fingerprints" of cone which characterizes its Geometry. Starting from spectrum one can re-construct the shape of junction;

[In acoustics, this problem was posed by Lord Rayleigh (1877), then advanced by Hermann Weyl (1911) and later by others,]

On Signals btwn domains with Variable Geometry

1. **Communication** between diverse dimension domains by wave signals with $m \neq 0$ **is impossible**.
2. In the 2-dim domain for real frequency ω there is a **total reflection on the cone** of the wave coming from $z = +\infty$, [accompanied by change of the scattered wave phase.]

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1. **Communication** between diverse dimension domains by wave signals with $m \neq 0$ **is impossible.**
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3. Physical conjunction of fields between differing geometry domains and of divers dimensions is a one more subtle item.

Puzzles of Conjunction

E.g., 4-vector field $A_{\mu=0,1,2,3}$ under DR, $D = 4 \rightarrow 2$ deforms into 2-component vector field $B_{\nu=0,1}$ and couple of scalar fields $\varphi_{\sigma=1,2}$. For the non-abelian gauge case, there appears nice interplay with gauge coupling, reduction scale and vector-scalar coupling.

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However, under the opposite, Kaluza-Klein, scenario with D2 unfolding to D4, one needs to admit a 2-component vector and 2 scalar fields living in the (1+1) prehistoric manifold with specific set of masses and coupling constants adequate for proper joining in the our beautiful (1+3)-world; combination prepared in advance by some script writer ...

In the spirit of the Antropic Principle ?

Tribute to Sacha Vasiliev

Scientific contacts
since late 70s,

Working together
on RG-conferences
since 1985

More than 20 years
of friendship

